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## Revised: Spring2009 Huseyin Yuce and Urmi Ghosh-Dastidar

1) The following are the scores of 20 students at New York City College of Technology: $222,233,254,241,251,268,276,220,238,253,249,236,247,256,261,227,257$, 244, 239, 242.
a) Construct a frequency distribution using 6 classes.
b) Draw a histogram for the frequency distribution in part (a).
c) Draw a frequency polygon for the frequency distribution in part (a).
2) There are six photocopying machines in a college office. During August 1999, these machines produce $2567,5456,3769,2245,6678$, and 3398 copies. Find the mean, median, and mode of the number of copies produced by these machines.
3) In a recent contest, the mean score was 210 and the standard deviation was 25 .
a) Find the z score of John who scored 190.
b) Find the z-score of Bill who scored 270.
c) If Mary had a z-score of 1.25 , what was Mary's score?
4) There is 1 red ball, 1 blue ball, and 1 yellow ball in a hat. Two balls are selected. Construct a tree diagram and list the sample space if the selection is done
a) with replacement.
b) without replacement.
5) A card is selected at random from a deck of 52 cards. What is the probability that this card is either a Queen or a Diamond?
6) A box contains 30 marbles: 15 red, 10 blue, and 5 green.
a) Two marbles are selected with replacement. Find the probability that both marbles are red.
b) Two marbles are selected without replacement. Find the probability of first selecting a blue marble then a green marble.
7) A study group is to be selected from 5 freshmen, 7 sophomores, and 4 juniors.
a) If a study group is to consist of 2 freshmen, 3 sophomores, and 1 junior, how many different ways can the study group be selected?
b) If a study group consisting of 6 students is selected, what is the probability that the group will consist of 2 freshmen, 3 sophomores, and 1 junior?
8) The following is the probability distribution of Jeanette's study hours for Mathematics in a given week.

| X (hours) | 2 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | 0.21 | 0.39 | $?$ | .15 |

a) Find the probability of $X=5$.
b) Find the mean of the probability distribution.
c) Find the standard deviation of the probability distribution.
9) Jonathan likes to play soccer. Assume $y=$ number of goals that Jonathan has scored during fall season and assume y can only take values from 0 to 4 . The following table represents the probability distribution for the discrete random variable $y$.

| y | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{y})$ | 0.11 | 0.28 | 0.29 | $?$ | 0.17 |

a) Find the probability that Jonathan would score exactly 3 goals in the fall season.
b) Find the probability that Jonathan would score at least 2 goals in fall season.
c) Find the mean of the probability distribution.
d) Find the standard deviation of the probability distribution.
10) According to the NIH, $32 \%$ of all women will fracture their hip by age 90 . If 8 women aged 90 are selected at random, what is the probability that exactly 5 of them will have suffered a hip fracture?
11) New York State has found that $30 \%$ of all consumer complaints are valid. If the State received 6000 complaints last year, about how many of them are expected to be valid? (i.e. find the mean). Find the standard deviation.
12) The number of major earthquakes in a year is approximately normally distributed with a mean of 20.8 and a standard deviation of 4.5 .
a) Find the probability that in a given year there will be less than 21 earthquakes.
b) Find the probability that in a given year there will be between 18 and 23 earthquakes.
13) A professor has found that the grades on the Statistics Final are normally distributed with a mean of 68 and a standard deviation of 15 . If only the best $14 \%$ of the grade in the class will receive an A, what grade must a student obtain in order to get an A ?
14) One tire manufacturer claims that his tires last an average of 42,000 miles with a standard deviation of 7800 miles. A random sample of 100 of his tires is taken. What is the probability that the average of these 100 tires will last greater than 41,000 miles?
15) It is claimed that the average annual per person spending on prescription drugs is $\$ 410$. If a survey of 65 randomly selected people indicated an average spending of $\$ 425$ with a standard deviation of $\$ 45$, do we reject the claim that the average is $\$ 410$ ? Use a $5 \%$ level of significance.
16) A survey claims that a college graduate from Smith College can expect an average starting salary of $\$ 42,000$. Fifteen Smith College graduates, drawn from normally distributed population, had an average starting salary of $\$ 40,800$ with a standard deviation of $\$ 2,250$. At the $1 \%$ level of significance, can we conclude that the average starting salary of the graduates is significantly less than $\$ 42,000$ ?
17) Given the following data:

| $\boldsymbol{X}$ | 1 | 4 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{Y}$ | 9 | 7 | 8 | 1 |

a) Find the coefficient of correlation.
b) Find the equation of the regression line.
c) What is the predicted value for $Y$ if $X=3$ ?
18) How many 5-person committees are possible from a group of 11 people if:
a) There are no restrictions
b) Both Jim and Mary must be on the committee
19) From a standard 52-card deck, what is the probability of 5-card hand having at least one face card?
20) How many 5-digit ZIP code numbers are possible if consecutive digits must be different?
21) The following data represent the number of days absent ( X ) and final grade ( Y ) for college students in a general education course.
a) Find the coefficient of correlation.
b) Find the equation of the regression line.
c) If one has 5 absences, what final grade will he/she get?
d) If one has 11 absences, what final grade will he/she get?

| $\mathbf{X}=$ Absences |  |
| :---: | :---: |
|  | $\mathbf{Y}=$ Final Grade |
| 2 | 83.5 |
| 5 | 73.9 |
| 7 | 71.8 |
| 3 | 81.1 |

$$
\begin{array}{ll}
9 & 66.2 \\
6 & 64.3 \\
4 & 78.2 \\
8 & 65.5 \\
0 & 89.2 \\
1 & 86.4
\end{array}
$$

22) The marks on a statistics exam are normally distributed with mean 70 and standard deviation of 10 .
a) What proportion of students will receive more than 80 ?
b) Find the probability that a mark will be between 60 and 90 .
c) If less than 60 is a failing grade, what is the probability that a student fails the class.
d) If only the best $10 \%$ of the grade in the class will receive an A, what grade must a student obtain in order to get an A ?
23) A multiple-choice exam has 8 questions, and each question has 3 possible answers. If you guess the answer to each question and don't leave any blank, what is the probability you get exactly 4 answers correct?
24) A true false test has six questions. If you use pure random guess to answer all questions, what is the percentage that you would have (a) exactly 3 correct answers?
(b) zero correct answer?
25) A pharmaceutical company manufactures a certain drug. The company must be certain that the standard deviation of the drug content in the tablet is not more than 0.1 mg . Twenty-five tablets are randomly selected and the amount of drug in each tablet is measured. The sample has a mean of 20 mg . and a variance of 0.015 mg . Does the data suggest at the level $\alpha=0.01$ that the standard deviation of drug content in the tablets is greater than 0.1 mg ?
26) One of the products produced by Branco Food Company is All-Bran Cereal, which competes with three other brands of similar all-bran cereals. The company's research office wants to investigate if the percentage of people who consume allbran cereal is the same for each of these four brands. Let us denote the four brands of cereal A, B, C, and D. A sample of 1000 persons who consume all-bran cereal was taken, and they were asked which brand they often consume. Of the respondents, 212 said they usually consume Brand A, 284 consume Brand B, 254 consume Brand C, and 250 consume Brand D. Does the sample provide enough evidence to reject null hypothesis that the percentage of people who consume allbran cereal is the same for all four brands? Use $\alpha=0.05$.

Answers:

1) a)
b)
c)

| Class | Frequency |
| :--- | :--- |
| $220-229$ | 3 |
| $230-239$ | 4 |
| $240-249$ | 5 |
| $250-259$ | 5 |
| $260-269$ | 2 |
| $270-279$ | 1 |



2) mean $=4018.83 \quad$ median $=3583.5 \quad$ mode $=$ none
3) a) -0.8
b) 2.4
c) 241.25
4) a)

b)

5) $\frac{4}{13}$
6) a) $\frac{1}{4}$
b) $\frac{5}{87}$
7) a) 1400 b) 0.1748
8) a) 0.25
b) $\mu=4.13$
c) $\sigma=1.29$
9) a) 0.15 b) 0.61 c) 1.99 d$) 1.24$
10) 0.059
11) $\mu=1800 \quad \sigma=35.50$
12) a) 0.516 b) 0.4203
13) 84.2
14) 0.8997
15) Reject $H_{0}: \mu=410, \quad H_{1}: \mu \neq 410$
16) Can't reject $H_{0}: \mu=42000, \quad H_{1}: \mu<42000($ critical value $=2.977, t=-2.07)$
17) a) $r=-0.7186$ b) $Y=10.642-0.976 X \quad$ c) 7.714
18) a) 462
b) 84
19) 0.7468
20) 65610
21) a) -0.94739
b) $Y=88.73273-2.82727 X$ c) 74.5963
d) 57.6327
22)
a) 0.158655
b) 0.818594
c) 0.158655
d) 82.8
23) 0.170705685
24) a) $0.3125=$ approximately $31 \%$ b) $0.0156=$ approximately $1.6 \%$
25) Reject $H_{0}: \mu=35, H_{1}: \mu \neq 35$ (critical value $=2.093, t=-8.94$ )
26) Critical value: $\chi^{2}=7.815$; test statistic : $\chi^{2}=10.464$; reject $H_{0}$.

