Self-Test :
Part A:

1) **a)** quotient: $x^2 + x + 1$; remainder 3

b) quotient $2x^3 - x^2 - 1$; remainder 0

c) quotient $x^3 - 2x^2 + x - 1$; remainder $-4x$
Note: you cannot use synthetic division for part (c).

2) substitute 1 for x: $1^{112} - 2(1^8) + 9(1^5) - 4(1^4) + 1 - 5 = 0$ so the remainder is 0: by the Remainder Theorem, which says that the remainder when you divide $f(x)$ by $x - c$ is $f(c)$.

3) Yes: either divide synthetically or use substitution as in #2 to show that the remainder is 0 when you divide by $x - 1$, so $x - 1$ is a factor. (You must show your work, of course)

4) You should get remainder 0; the other factor (the quotient) is $x^5 - 3x^4 + 2x^3 + 5x^2 - 7x + 2$

5) The roots are

1, with multiplicity 3

3, with multiplicity 2

-3, with multiplicity 2

-2, with multiplicity 1

Part B:

1) **a)** $f \circ g(x) = f\left(\frac{x}{3} + 4\right) = x$ (Show all work)

b) $g \circ f(x) = g(3x - 12) = x$ (Show all work)

c) Yes, because $f(x)$ and $g(x)$ satisfy the "Round-trip" Theorem, by (a) and (b).

2) The inverse is $f^{-1}(x) = -\frac{x}{3} + \frac{5}{3}$, or $\frac{5-x}{3}$

3) f(x) is not one-to-one, because the graph fails the horizontal line test. There are places where a horizontal line will intersect the graph in more than one point. (You should draw a horizontal line which intersects the graph in more than one point to show this.)

4) $f(x) = -(x + 5)^2 + 3$

Part C:

F1a) Real roots: $-3, \frac{1}{2}$

F1b) $2\left(x - \frac{1}{2}\right)(x + 3)(x - 2i)(x + 2i) = (2x - 1)(x + 3)(x - 2i)(x + 2i)$ (You could leave it in the first form.)

F2) Hint: if $i - 3$ is a root, then so is its conjugate $-i - 3$.
$f(x) = -3(x - 1)(x - i + 3)(x + i + 3)$

F3a) The degree is odd (because of the end behavior)

F3b) The smallest possible degree is 5 (because there are (at least) 5 roots shown in the graph: note the apparently double root)

F3c) The roots are 1 (multiplicity 1), 2 (multiplicity 1), 3 (multiplicity 1), and 4 (multiplicity 2).