For more information and to discuss these problems, go to our OpenLab course!

Instructions: These problems are for you to use to test yourself, **after** you have practiced with the routine homework assignments, to see how ready you are for Test 2. They are not meant as a substitute for regular and diligent practice!

Do the following problems as if you were taking a test: without notes or textbook, and give yourself a time limit as stated at the start of each self-test. At the end of that time, check your answers in the back of the textbook: the page number is given next to the problem. Then review as needed before you repeat the self-test.

Self-Test : allow 50 minutes for parts A-B

Part A:

1) State the quotient and remainder when the first polynomial is divided by the second.

a) $x^3 + 5x^2 + 5x + 7$; $x + 4$

b) $2x^4 - 5x^3 + 2x^2 - x + 2$; $x - 2$

c) $x^5 - 2x^3 - 2x^4 + 5x^2 - 7x + 3$; $x^2 - 3$

2) What is the remainder when $x^{112} - 2x^8 + 9x^5 - 4x^4 + x - 5$ is divided by $x - 1$? Find it without using long division or synthetic division, and explain your reasoning.

3) Is $x - 1$ a factor of $f(x) = 14x^{87} - 65x^{56} + 51$? Justify your answer.

4) Use synthetic division to show that $x - 2$ is a factor of $x^6 - 5x^5 + 8x^4 + x^3 - 17x^2 + 16x - 4$ and find the other factor.

5) List the roots of the following polynomial and their multiplicities:

$$f(x) = 3(x - 1)^3(x^2 - 9)^2(x + 2)$$

Part B:

1) For the functions $f(x) = 3x - 12$ and $g(x) = \frac{x}{3} + 4$, find each of the following:

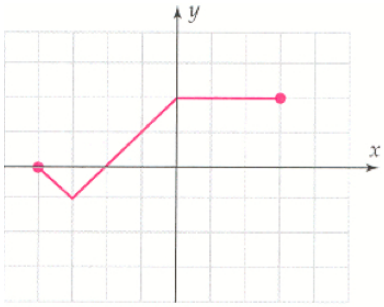
a) $f \circ g(x) =$

b) $g \circ f(x) =$

c) Is g the inverse of f? Explain.

2) Use algebra to find the inverse of $f(x) = -3x + 5$

3) The graph of a function $f(x)$ is given. Is $f(x)$ one-to-one? Explain.



4) Write the rule for a function f whose graph can be obtained from the graph of $g(x) = x^2$ by reflecting in the x-axis, shifting the graph vertically up 3 units, then shifting horizontally to the left 5 units.

Part C: allow 30 minutes

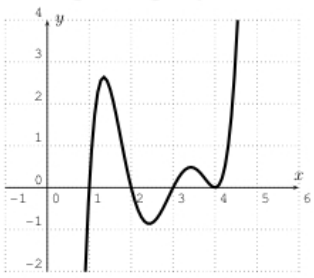
1a) Find all the real roots of the polynomial

$$2x^4 + 5x^3 + 5x^2 + 20x - 12$$

F1b) Factor the polynomial in part (a) completely over the complex numbers.

F2) Find a polynomial $f(x)$ that has the following properties: f has degree 3, the coefficients of f are all real, f has roots 1 and $i - 3$, and $f(0)= 30$

F3) The complete graph of a polynomial is given below.



a) Is the degree of the polynomial even or odd?
b) What is the smallest possible degree of this polynomial?
c) If the polynomial has the degree you gave in (b), find the roots of the polynomial and their multiplicities.