# MAT 2580 Test 1 Practice Questions Spring 2012

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1. Add the following matrices:

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} + \begin{pmatrix} d & 0 \\ e & f \end{pmatrix}$$

2. For

$$A = \begin{pmatrix} 13 & 24 & 56 & 78 \\ 91 & -2 & -34 & -56 \\ -78 & -89 & 10 & 23 \end{pmatrix},$$

add $A + 0_{3,4}$.

3. Multiply:

$$2\begin{pmatrix} 1 & 0 & 2 \\ 4 & 3 & 0 \\ 0 & -4 & -1 \end{pmatrix}.$$

4. For

$$B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ 0 & -1 & -2 & -3 \\ -2 & -1 & 0 & 1 \\ 2 & 3 & 4 & 5 \end{pmatrix},$$

compute $B - 4I_4$.

5. Multiply:

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} I_2.$$

6. Multiply:

$$\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix}.$$

7. Multiply

$$\begin{pmatrix} 2 & b \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & c \\ 0 & -2 \end{pmatrix}.$$

8. Compute
$$\begin{pmatrix} 2 & b \\ 0 & 3 \end{pmatrix}^T.$$

9. Find the length of $\begin{pmatrix} -3 \\ 4 \\ -12 \end{pmatrix}$.

10. Build a 0-1 matrix for the relation on $[2, 3, 4, 5, 6] \times [2, 3, 4, 5, 6, 8, 9, 10]$, $xRy$ iff $gcd(x, y) = 1$.

11. For $L(x_1, x_2) = (y_1, y_2)$ if and only if $A\left(\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right) = \left(\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right)$ with
$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$
evaluate $L(-3, 4)$.

12. For $Q(\overrightarrow{v}) = y$ if and only if $\overrightarrow{v}^{\mathrm{T}} A \overrightarrow{v} = (y)$ with
$$G = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix},$$
evaluate $Q\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right)$.

13. For the following function, $f$, from $[a, b, c, d, e]$ to [“vowel”, “consonant”],
$$A \mapsto \text{“vowel”},$$
$$B \mapsto \text{“consonant”},$$
$$C \mapsto \text{“consonant”},$$
$$D \mapsto \text{“consonant”},$$
$$E \mapsto \text{“vowel”},$$
build the 0-1 matrix that represents $f$.

14. Mark which of the following $0 - 1$ matrices represents a finite relation which are functions from $x$ to

$y$, which are onto functions from $x$ to $y$ and which are 1-to-1 functions from $x$ to $y$. Here the relation $xRy$ is true iff $\overrightarrow{v}_y{}^{\mathrm{T}} C \overrightarrow{v}_x = (1)$.

| $C$ | is a function | is onto | is 1-to-1 |
|---|---|---|---|
| $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | | | |
| $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | | | |
| $\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$ | | | |
| $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | | | |

15. Use elemetary row operations to complete the forward phase of the row reduction process on this matrix to produce a matrix in row echelon form:

$$\begin{pmatrix} 0 & 1 & 2 & 4 \\ 3 & -2 & -1 & -5 \\ -3 & 1 & 0 & 0 \end{pmatrix}.$$

16. Use elemetary row operations to complete the backward phase of the row reduction process on this matrix to produce a matrix in row echelon form:

$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -2 & -1 & -5 \\ 0 & 0 & -1 & 2 \end{pmatrix}.$$

17. Of the matrices below, please indicate which are

square($\square$), lower triangular($\triangle$), upper triangular($\triangledown$), diagonal($\backslash$), symmetric(`sym`), in row-echelon form(`ref`), or in reduced row echelon form(`rref`). Here $B$ is as in number 4.

| D | ($\square$) | ($\triangle$) | ($\triangledown$) | ($\backslash$) | (`sym`) | (`ref`) | (`rref`) |
|---|---|---|---|---|---|---|---|
| $0_{2,3}$ | | | | | | | |
| $I_5$ | | | | | | | |
| $B - B^{\mathrm{T}}$ | | | | | | | |
| $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ | | | | | | | |
| $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ | | | | | | | |
| $\begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$ | | | | | | | |
| $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ | | | | | | | |
| $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | | | | | | | |

18. Please construct the $4 \times 4$ elementary matrix for the operation that swaps the second and fourth rows of a matrix when multiplying it on the left.

**Extra Credit:**

i. Prove that the sum of a square matrix and its transpose is symmetric.

ii. Find the flaw in this "proof" that the product of two symmetric matrices is itself symmetric. Note that

$$\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}.$$

**Claim 1** (False Statement)**.** *The product of two $n \times n$ symmetric matrices is itself symmetric.*

*Proof.* Let $S_1$ and $S_2$ represent two symmetric $n \times n$ matrices. Every symmetric matrix is the sum of a unique upper triagular matrix and its transpose. Let the corresponding upper triagular matrix for $S_i$ be denoted $T_i$. Here is the calculation.

$$\begin{aligned}
(S_1S_2)^{\mathrm{T}} &= ((T_1 + T_1^{\mathrm{T}})(T_2 + T_2^{\mathrm{T}}))^{\mathrm{T}} \\
&= (T_1T_2 + T_1^{\mathrm{T}}T_2 + T_1T_2^{\mathrm{T}} + T_1^{\mathrm{T}}T_2^{\mathrm{T}})^{\mathrm{T}} \\
&= (T_1T_2)^{\mathrm{T}} + (T_1^{\mathrm{T}}T_2)^{\mathrm{T}} + (T_1T_2^{\mathrm{T}})^{\mathrm{T}} + (T_1^{\mathrm{T}}T_2^{\mathrm{T}})^{\mathrm{T}} \\
&= T_2^{\mathrm{T}}T_1^{\mathrm{T}} + T_2^{\mathrm{T}}T_1 + T_2T_1^{\mathrm{T}} + T_2T_1 \\
&= (T_2^{\mathrm{T}}T_1^{\mathrm{T}} + T_2^{\mathrm{T}}T_1) + (T_2T_1 + T_2T_1^{\mathrm{T}}) \\
&= (T_2^{\mathrm{T}}T_1^{\mathrm{T}} + T_2^{\mathrm{T}}T_1) + (T_2^{\mathrm{T}}T_1^{\mathrm{T}} + T_2^{\mathrm{T}}T_1)^{\mathrm{T}}
\end{aligned}$$

This shows that the product is a sum of a matrix and its transpose and is therefore symmetric. □