MAT1475, Fall 2024 **Exam #2**
Instructor: Suman Ganguli **Monday, November 18** **Name:** ____________________

Question:	1	2	3	4	Total
Points:	15	10	15	10	50
Score:					

1. (15 points) Find the derivatives of the following functions, using the various differentiation rules:

(a) $f(x) = (3x-2)^5$

Solution:

$$f'(x) = 5(3x-2)^4 \cdot (3) = 15(3x-2)^4$$

(b) $y = \sin(1-3t^2)$

Solution:

$$\frac{dy}{dt} = \cos(1-3t^2) \cdot (-6t) = -6t\cos(1-3t^2)$$

(c) $P(x) = e^x \cdot \tan(2x)$

Solution:

$$\frac{dP}{dx} = e^x \cdot \tan(2x) + 2e^x \cdot \sec^2(2x)$$

2. (10 points) The volume of a sphere with radius r is given by the function $V(r) = \frac{4}{3}\pi r^3$

Note: leave all your answers below in terms of π, i.e., do *not* use a calculator to give decimal approximations.

(a) Calculate the volume of a sphere which has a radius of 2 cm, i.e., calculate $V(2)$

Solution:

$$V(2) = \frac{4}{3}\pi \cdot 2^3 = \frac{32}{3}\pi$$

(b) Find $V'(r)$, i.e., $\frac{dV}{dr}$:

Solution:

$$V'(r) = 4\pi r^2$$

(c) Calculate the rate of change of the volume when $r = 2$, i.e., calculate $V'(2)$:

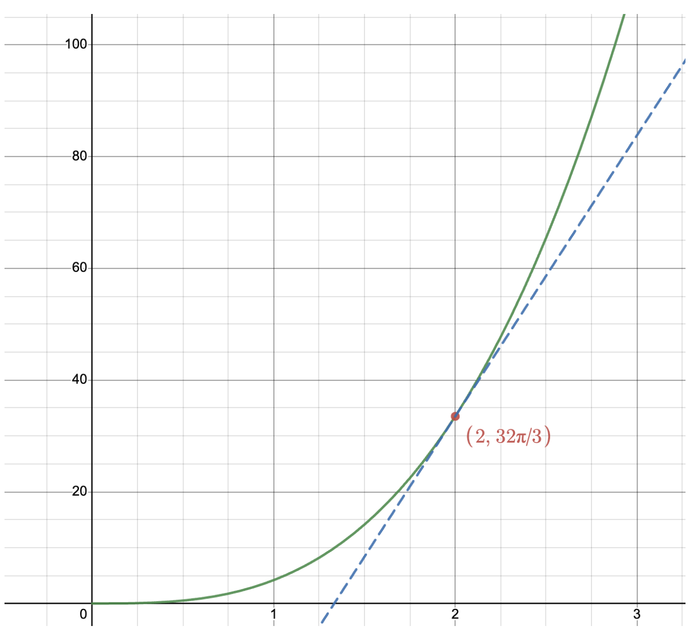
Solution:

$$V'(2) = 4\pi(2^2) = 16\pi$$

(d) Shown below is the graph of $y = V(r)$. Sketch the tangent line to the curve at $r = 2$, and write down the equation of that tangent line in point-slope form:

Solution: The equation of the tangent line at $r = 2$ is $y = V(2) + V'(2)(r - 2)$, i.e.

$$y = \frac{32\pi}{3} + 16\pi(r - 2)$$



3. (15 points) Find the derivatives of the following functions, using the various differentiation rules:

(a) $f(x) = \dfrac{\ln x}{x}$

Solution:

$$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

(b) $y = e^{x^3+1}$

Solution:

$$\frac{dy}{dt} = e^{x^3+1} \cdot (3x^2) = 3x^2 e^{x^3+1}$$

(c) $h(x) = \sin(\sqrt{2x+1})$)

Solution:

$$\frac{dh}{dx} = \cos(\sqrt{2x+1}) \cdot \frac{d}{dx}\left(\sqrt{2x+1}\right) = \cos(\sqrt{2x+1}) \cdot \frac{1}{2\sqrt{2x+1}} \cdot \frac{d}{dx}(2x+1) = \frac{\cos(\sqrt{2x+1})}{\sqrt{2x+1}}$$

4. (10 points) Consider the function $f(x) = \dfrac{6x-8}{x^2+4}$

(a) Calculate $f(0)$:

Solution: $f(0) = \dfrac{6(0)-8}{0^2+4} = -\dfrac{8}{4} = -2$

(b) Find the derivative of this function, using the Quotient Rule (you don't need to simplify your solution):

Solution: $f'(x) = \dfrac{(6)(x^2+4)-(6x-8)(2x)}{(x^2+4)^2}$

(c) Calculate $f'(0)$:

Solution: $f'(0) = \dfrac{(6)(0^2+4)-(0-8)(0)}{(0^2+4)^2} = \dfrac{24}{16} = \dfrac{3}{2}$

(d) Write the equation of the tangent line for the graph $y = f(x)$ at $(0, f(0))$:

Solution:

$$y = -2 + \frac{3}{2}x$$

(e) Shown below is the graph of $y = \dfrac{6x-8}{x^2+4}$. Sketch the tangent line whose equation you found in part (c):