**MAT1475, Fall 2024** **Quiz #2**
**Instructor: Suman Ganguli** **Wednesday, September 25** **Name:** ______________________

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Consider the function $f(x) = x^2 - 5x + 4$. Let's calculate $f'(a)$ using the limit definition of the derivative:

1. (4 points) Write down and simplify $f(a+h)$. Use that to write down and simplify $f(a+h) - f(a)$.

**Solution:**

$$f(a+h) = (a+h)^2 - 5(a+h) + 4 = a^2 + 2ah + h^2 - 5a - 5h + 4$$

$$f(a+h) - f(a) = (a^2 + 2ah + h^2 - 5a - 5h + 4) - (a^2 - 5a + 4) = 2ah + h^2 - 5h$$

2. (4 points) Use your answer from #1 to write down and simplify the difference quotient that appears in the definition of the derivative:

**Solution:**

$$\frac{\Delta f}{\Delta x} = \frac{f(a+h) - f(a)}{h} = \frac{2ah + h^2 - 5h}{h} = \frac{h(2a + h - 5)}{h} = 2a + h - 5$$

3. (2 points) Use the simplified difference quotient from #2 to calculate $f'(a)$:

**Solution:**

$$f'(a) = \lim_{h \to 0} (2a + h - 5) = 2a - 5$$

Note that if we replace $a$ (a symbol representing an arbitrary constant) by the variable $x$, we have shown, using the limit definition of the derivative, that for $f(x) = x^2 - 5x + 4$, its derivative is $f'(x) = 2x - 5$.

Of course, the derivative of a polynomial such as $f(x)$ is very easy to calculate using the differentiation rules (the Power Rule, together with the Sum/Difference, Constant Multiple, and Constant Rules). We can differentiate $f$ using the rules and check our derivation above:

$$f'(x) = \frac{df}{dx} = \frac{d}{dx}(x^2 - 5x + 4) = \frac{d}{dx}(x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(4) = 2x - 5 + 0 = 2x - 5$$