New York City College of Technology
Department of Mathematics

MAT 1275 Final Exam Review Problems[1]

1. Solve for x and simplify the answer.

(a) $6x^2 - 2x - 3 = 0$

(b) $-2x^2 + 4x - 5 = 0$

(c) $2x^2 + 2x = -3$

2. Given the following quadratic equation, state the x- and y-intercepts (solutions of the form $(x, 0)$ and $(0, y)$), the vertex, and then use this information to sketch the graph.

(a) $y = x^2 - 2x - 3$

(b) $y = x^2 + 2x$

(c) $y = -x^2 + 4x - 2$

3. Simplify the complex fraction.

(a) $\dfrac{\dfrac{2}{y^2} + \dfrac{1}{y}}{\dfrac{4}{y^2} - \dfrac{1}{y}}$

(b) $\dfrac{\dfrac{2}{x} + \dfrac{1}{y}}{\dfrac{3}{y} - \dfrac{4}{x}}$

(c) $\dfrac{3 - \dfrac{1}{2x}}{2 + \dfrac{1}{x^2}}$

4. Write the equation of the circle given below in standard form. Identify the center and radius of the circle and graph it. Label four points on the graph with coordinates.

(a) $x^2 + y^2 - 6x + 4y - 4 = 0$

(b) $x^2 + y^2 + 8x - 2y - 8 = 0$

(c) $x^2 + y^2 + 10x + 6y + 18 = 0$

5. Perform the indicated operation and express the answer in the form $a + bi$.

(a) $(-2 + 3i)(5 - 7i)$

(b) $(4 - 9i)(3 - 2i)$

(c) $\dfrac{3 - 2i}{1 - i}$

(d) $\dfrac{2 - 7i}{3 + 4i}$

6. Solve the system of equations.

[1]Revised by Professors Carley, Colucci, Kan, and Rozenblyum in Fall 2019.

(a) $\begin{cases} 3x + 2y - z = 4 \\ 2x - y + 2z = 10 \\ x + 3y - 4z = -7 \end{cases}$

(b) $\begin{cases} -2x + 5y + z = 8 \\ x - 2y - 3z = -13 \\ x + 3y - z = 5 \end{cases}$

(c) $\begin{cases} 2x - 3y + z = -9 \\ 3x + 5y + 2z = 16 \\ -4x + 2y - 3z = 4 \end{cases}$

7. Solve the system of equations.

(a) $\begin{cases} 3x + y = 4 \\ x^2 - 3y = -32 \end{cases}$

(b) $\begin{cases} x^2 + y^2 = 5 \\ x - y^2 = -3 \end{cases}$

(c) $\begin{cases} 4x - y = 2 \\ x^2 + 2y = 5 \end{cases}$

(d) $\begin{cases} x^2 - y^2 = 3 \\ 2x + y^2 = 5 \end{cases}$

8. An airplane is flying 700 feet above ground. From the plane to the base of the control tower, the angle of depression is 47°. How far away is ground directly underneath the plane to the control tower? Round your answer to the nearest tenth.

9. A temporary ramp that is 15 feet long is placed to reach an entrance door that is 4 feet above the ground. What is the angle of elevation of the ramp? Round your answer to the nearest tenth.

10. A wire, bolted to the ground 7 feet away from the base, is helping a tree stay upright. If the angle of elevation of the wire is 44°, how long is the wire? Round your answer to the nearest tenth.

11. For the given expression,

1. identify the quadrant in which the angle is located,
2. find the reference angle,
3. calculate the exact value.

(a) $\tan(-135°)$

(b) $\cos(240°)$

(c) $\sin(330°)$

(d) $\cos(-120°)$

(e) $\cot\left(-\frac{5\pi}{4}\right)$

(f) $\csc\left(-\frac{4\pi}{3}\right)$

(g) $\sec\left(\frac{2\pi}{3}\right)$

12. For the given information, state the value of the five remaining trig functions of θ.

(a) $\sin\theta = \dfrac{2}{5}$ and $\cos\theta < 0$

(b) $\tan\theta = 2$ and $\sin\theta < 0$

(c) $\cos\theta = \dfrac{2}{3}$ and $\tan\theta < 0$

(d) $\cos\theta = \dfrac{3}{7}$ and $\sin\theta < 0$

13. Given a ΔABC, answer the following. Round each answer to the nearest tenth.

(a) If $a = 12$, $b = 8$ and $c = 5$, find $\angle C$.

(b) If $\angle A = 50°$, $\angle B = 75°$ and $a = 20$, find side b.

(c) If $b = 9$, $c = 6$ and $\angle A = 67°$, find side a.

(d) If $\angle C = 37°$, $\angle B = 79°$ and $b = 13$, find side a.

14. Prove the identity.

(a) $\cos x + \sin x \cdot \tan x = \sec x$

(b) $\csc x - \sin x = \cot x \cdot \cos x$

(c) $\dfrac{1}{\cos x} - \dfrac{1}{\sec x} = \tan x \cdot \sin x$

(d) $\csc x \cdot \cot x \cdot \sec x = 1 + \cot^2 x$

(e) $\sin x \cdot \tan x \cdot \cos x = 1 - \cos^2 x$

(f) $\sec x \cdot \csc x = \tan x + \cot x$

15. Find the exact solutions to the trig equation for $x \in [0, 2\pi)$.

(a) $4\sin x + 2 = 0$

(b) $6\cos x - 3 = 0$

(c) $2\tan x - 2 = 0$

(d) $6\tan x = -2\sqrt{3}$

16. Solve for x and round the answer to the nearest tenth.

(a) $3^x = 38$

(b) $2^x = 19$

(c) $4^x = 7$

17. Evaluate the logarithm without using a calculator.

(a) $\log_2 \sqrt[3]{2}$

(b) $\log_3 \dfrac{1}{9}$

(c) $\log_4(16\sqrt[3]{4})$

(d) $\log_2(8\sqrt[6]{2})$

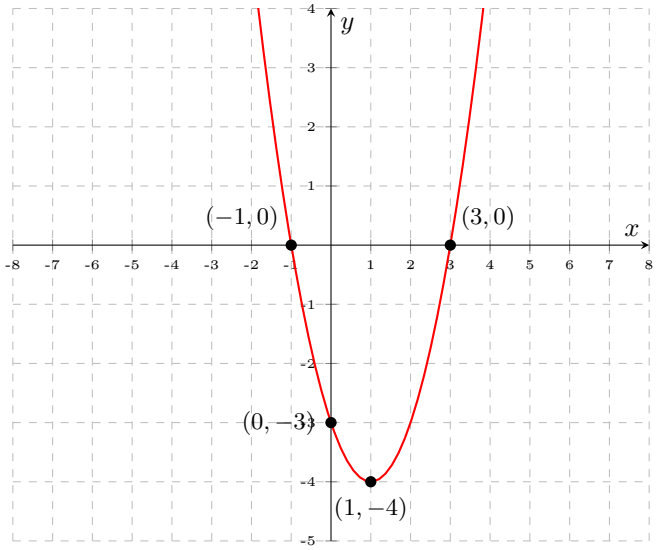
Answers:

1. **(a)** $\dfrac{1 \pm \sqrt{19}}{6}$

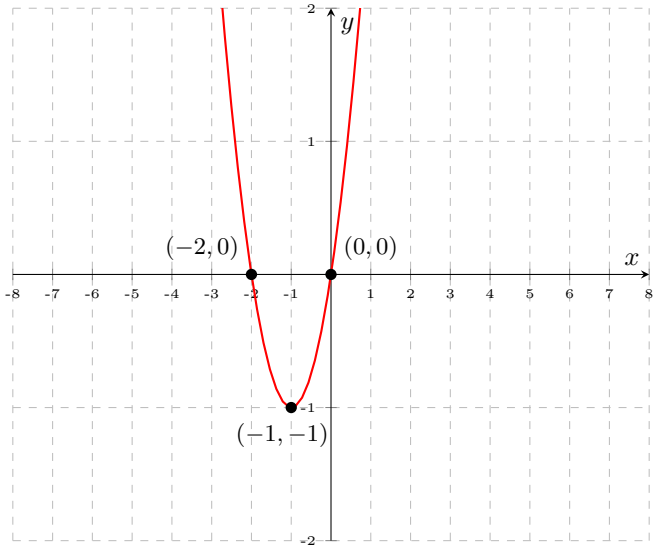
(b) $1 \pm \dfrac{\sqrt{6}}{2}i$

(c) $-\frac{1}{2} \pm \frac{\sqrt{5}}{2}i$

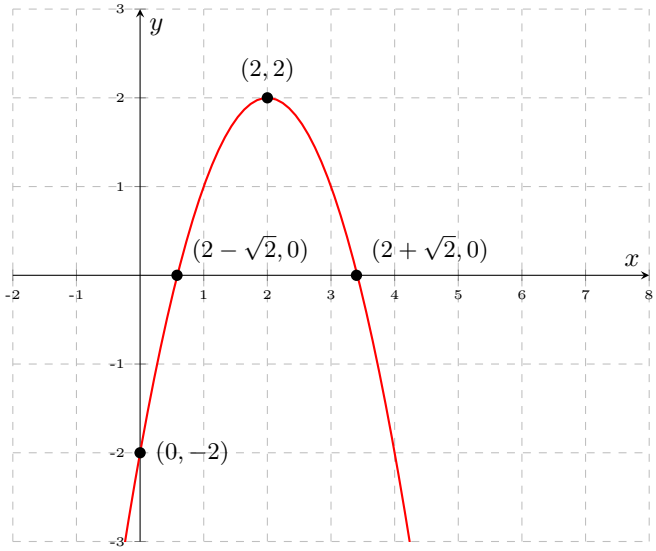
2. **(a)** Vertex: $(1, -4)$, x-intercepts: $(3, 0)$ and $(-1, 0)$, y-intercept: $(0, -3)$



(b) Vertex: $(-1, -1)$, x-intercepts: $(0, 0)$ and $(-2, 0)$, y-intercept: $(0, 0)$



(c) Vertex: $(2, 2)$, x-intercepts: $(2 - \sqrt{2}, 0)$ and $(2 + \sqrt{2}, 0)$, y-intercept: $(0, -2)$

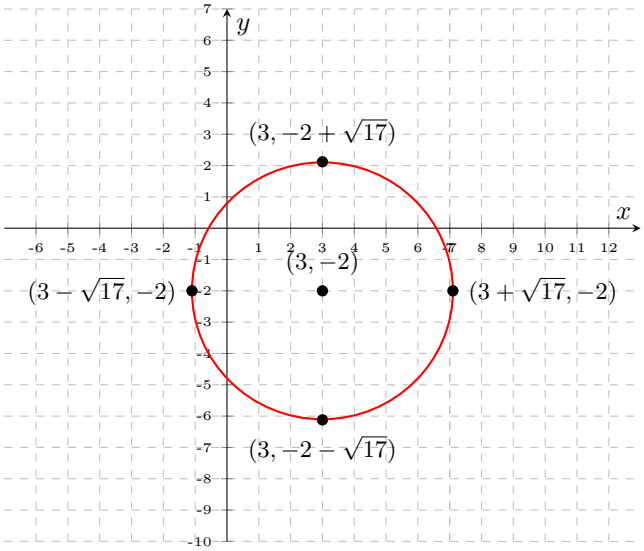


3. **(a)** $\dfrac{2+y}{4-y}$

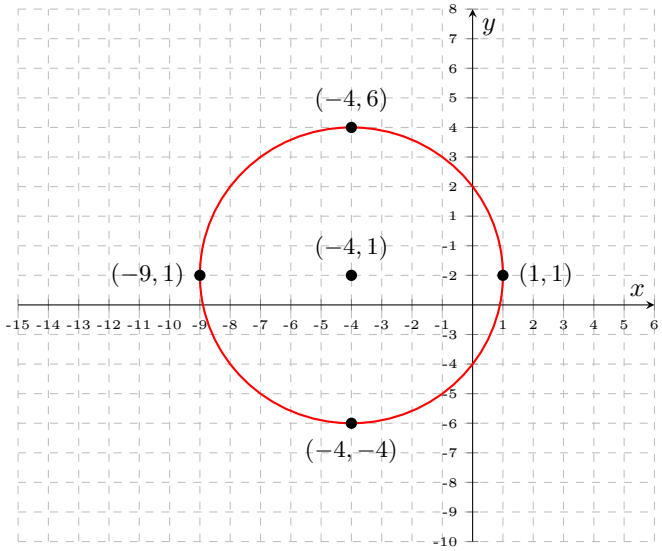
(b) $\dfrac{2y+x}{3x-4y}$

(c) $\dfrac{6x^2-x}{4x^2+2}$ or $\dfrac{x(6x-1)}{2(2x^2+1)}$

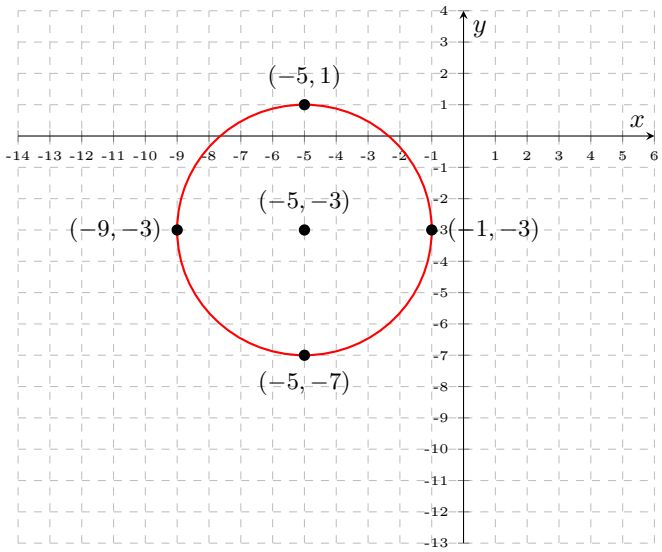
4. **(a)** Center: $(3,-2)$, radius: $r=\sqrt{17}$



(b) Center: $(-4,1)$, radius: $r=5$



(c) Center: $(-5, -3)$, radius: $r = 4$



5. **(a)** $11 + 29i$

(b) $-6 - 35i$

(c) $\frac{5}{2} + \frac{1}{2}i$

(d) $-\frac{22}{25} - \frac{29}{25}i$

6. **(a)** $x = 3,\ y = -2,\ z = 1$

(b) $x = 3,\ y = 2,\ z = 4$

(c) $x = -1,\ y = 3,\ z = 2$

7. **(a)** $(-5, 19),\ (-4, 16)$

(b) $(-2, 1),\ (1, 2),\ (1, -2),\ (-2, -1)$

(c) $(-9, -38),\ (1, 2)$

(d) $(2, 1),\ (2, -1),\ (-4, \sqrt{13}),\ (-4, -\sqrt{13})$

8. 652.8 ft

9. 15.5°

10. 9.7 ft

11. **(a)** III, 45°, 1

(b) III, 60°, $-\frac{1}{2}$

(c) IV, 30°, $-\frac{1}{2}$

(d) III, 60°, $-\frac{1}{2}$

(e) II, $\frac{\pi}{4}$, -1

(f) II, $\frac{\pi}{3}$, $\frac{2\sqrt{3}}{3}$

(g) II, $\frac{\pi}{3}$, -2

12. **(a)** $\cos\theta = -\frac{\sqrt{21}}{5}$, $\tan\theta = -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$, $\sec\theta = -\frac{5}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$, $\csc\theta = \frac{5}{2}$, $\cot\theta = -\frac{\sqrt{21}}{2}$

(b) $\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$, $\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$, $\sec\theta = -\sqrt{5}$, $\csc\theta = -\frac{\sqrt{5}}{2}$, $\cot\theta = \frac{1}{2}$

(c) $\sin\theta = -\frac{\sqrt{5}}{3}$, $\tan\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = \frac{3}{2}$, $\csc\theta = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$, $\cot\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

(d) $\sin\theta = -\frac{2\sqrt{10}}{7}$, $\tan\theta = -\frac{2\sqrt{10}}{3}$, $\sec\theta = \frac{7}{3}$, $\csc\theta = -\frac{7}{2\sqrt{10}} = -\frac{7\sqrt{10}}{20}$,
$\cot\theta = -\frac{3}{2\sqrt{10}} = -\frac{3\sqrt{10}}{20}$

13. **(a)** 17.6°

(b) 25.2

(c) 8.6

(d) 11.9

14. **(a)**

$$\begin{aligned}
\cos x + \sin x \cdot \tan x &= \cos x + \sin x \cdot \frac{\sin x}{\cos x} \\
&= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos x} \\
&= \frac{1}{\cos x} \\
&= \sec x
\end{aligned}$$

(b)

$$\begin{aligned}\csc x - \sin x &= \frac{1}{\sin x} - \sin x \\ &= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \\ &= \frac{1 - \sin^2 x}{\sin x} \\ &= \frac{\cos^2 x}{\sin x} \\ &= \frac{\cos x}{\sin x} \cdot \cos x \\ &= \cot x \cdot \cos x\end{aligned}$$

(c)

$$\begin{aligned}\frac{1}{\cos x} - \frac{1}{\sec x} &= \frac{1}{\cos x} - \cos x \\ &= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} \\ &= \frac{1 - \cos^2 x}{\cos x} \\ &= \frac{\sin^2 x}{\cos x} \\ &= \frac{\sin x}{\cos x} \cdot \sin x \\ &= \tan x \cdot \sin x\end{aligned}$$

(d)

$$\begin{aligned}\csc x \cdot \cot x \cdot \sec x &= \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} \\ &= \frac{1}{\sin^2 x} \\ &= \csc^2 x \\ &= 1 + \cot^2 x\end{aligned}$$

(e)

$$\begin{aligned}\sin x \cdot \tan x \cdot \cos x &= \sin x \cdot \frac{\sin x}{\cos x} \cdot \cos x \\ &= \sin^2 x \\ &= 1 - \cos^2 x\end{aligned}$$

(f) For this identity, it is more convenient to start from the right hand side.

$$\begin{aligned}\tan x + \cot x &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x} \\ &= \frac{1}{\cos x \cdot \sin x} \\ &= \frac{1}{\cos x} \cdot \frac{1}{\sin x} \\ &= \sec x \cdot \csc x\end{aligned}$$

15. **(a)** $\frac{7\pi}{6}, \frac{11\pi}{6}$

(b) $\frac{\pi}{3}, \frac{5\pi}{3}$

(c) $\frac{\pi}{4}, \frac{5\pi}{4}$

(d) $\frac{5\pi}{6}, \frac{11\pi}{6}$

16. **(a)** $x = 3.3$

(b) $x = 4.2$

(c) $x = 1.4$

17. **(a)** $\frac{1}{3}$

(b) -2

(c) $\frac{7}{3}$

(d) $\frac{19}{6}$