MAT1475, Fall 2023 | **Quiz #1**
Instructor: Suman Ganguli | **Wednesday, September 20** | **Name:** ____________________

Consider the function $f(x) = x^2 + 3x + 2$. Let's calculate $f'(1)$ using the limit definition of the derivative:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

1. (1 point) What is $f(1)$? Show your calculation:

> **Solution:** $f(1) = 1^2 + 3(1) + 2 = 1 + 3 + 2 = 6$

2. (3 points) Write down and simplify the difference quotient that appears in the definition of the derivative.
(Hint: substitute the given expression for $f(x)$ and the value of $f(1)$ you calculated; then combine the constant terms in the numerator and factor the resulting quadratic polynomial.)

> **Solution:**
> $$\frac{\Delta f}{\Delta x} = \frac{f(x) - f(1)}{x - 1} = \frac{(x^2 + 3x + 2) - 6}{x - 1} = \frac{x^2 + 3x - 4}{x - 1} = \frac{(x + 4)(x - 1)}{x - 1} = x + 4$$

3. (2 points) Use the simplified difference quotient to calculate $f'(1)$:

> **Solution:**
> $$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} (x + 4) = 1 + 4 = 5$$

4. (2 points) Use the point-slope form to write down the equation of the tangent line passing through $(1, f(1))$:

> **Solution:** The point on the graph is $(1, 6)$ and the slope of the tangent line is $f'(1) = 5$. Hence, according to point-slope form ($y = y_0 + m(x - x_0)$), the equation of the tangent line is:
> $$y = 6 + 5(x - 1)$$

5. (2 points) Shown below is the graph of $y = f(x) = x^2 + 3x + 2$. Label the point $(1, f(1))$ on the graph and sketch the tangent line to the curve passing through that point: