MAT1475, Fall 2023 **Exam #2**
Instructor: Suman Ganguli **Monday, November 20** **Name:** ____________________

Question:	1	2	3	4	Total
Points:	10	15	15	10	50
Score:					

1. (10 points) The volume of a sphere with radius r is given by the function $V(r) = \frac{4}{3}\pi r^3$:

(a) Calculate the volume of a sphere which has a radius of 2 cm:

Solution:

$$V(2) = \frac{4}{3}\pi 2^3 = \frac{32}{3}\pi$$

(b) Find $V'(r)$, i.e., $\frac{dV}{dr}$:

Solution:

$$V'(r) = 4\pi r^2$$

(c) Calculate the rate of change of the volume when $r = 2$:

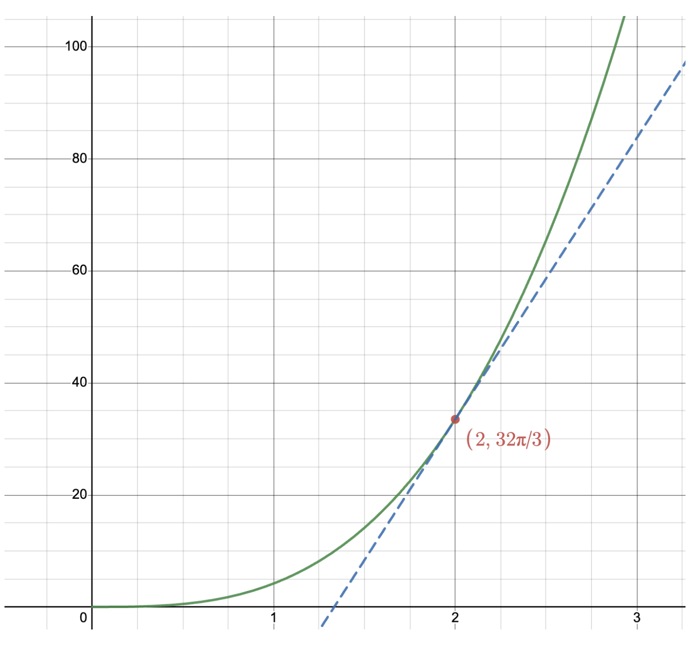
Solution:

$$V'(2) = 4\pi(2^2) = 16\pi$$

(d) Shown below is the graph of $y = V(r)$. Sketch the tangent line to the curve at $r = 2$, and write down the equation of that tangent line in point-slope form:

Solution: The equation of the tangent line at $r = 2$ is $y = V(2) + V'(2)(r - 2)$, i.e.

$$y = \frac{32\pi}{3} + 16\pi(r - 2)$$



2. (15 points) Find the derivatives of the following functions, using the various differentiation rules:

(a) $f(x) = (3x-2)^5$

Solution:

$$f'(x) = 5(3x-2)^4 \cdot (3) = 15(3x-2)^4$$

(b) $y = 2\cos(\pi t)$

Solution:

$$\frac{dy}{dt} = -2\sin(\pi t) \cdot \pi = -2\pi \sin(\pi t)$$

(c) $P(t) = e^x \cdot \tan(2x)$

Solution:

$$\frac{dP}{dt} = e^x \cdot \tan(2x) + 2e^x \cdot \sec^2(2x)$$

(d) $h(x) = \sin(\sqrt{2x+1}))$

Solution:

$$\frac{dh}{dx} = \cos(\sqrt{2x+1}) \cdot \frac{d}{dx}\left(\sqrt{2x+1}\right) = \cos(\sqrt{2x+1}) \cdot \frac{1}{2\sqrt{2x+1}} \cdot \frac{d}{dx}(2x+1) = \frac{\cos(\sqrt{2x+1})}{\sqrt{2x+1}}$$

3. (15 points) Suppose a cannonball is launched straight up at time $t = 0$, such that its height (in meters) at time t seconds is given by the function

$$h(t) = -t^2 + 8t + 20 \quad (t \geq 0)$$

(a) Write down the velocity and acceleration functions of the cannonball:

Solution:

$$v(t) = h'(t) = -2t + 8$$

$$a(t) = h''(t) = -2$$

(b) From what height was the cannonball launched (i.e., what is the initial height of the cannonball)? What is the initial velocity of the cannonball? Show your calculations, and include units with your answers.

Solution:

$$h(0) = 20\, m$$

$$v(0) = 8\, m/s$$

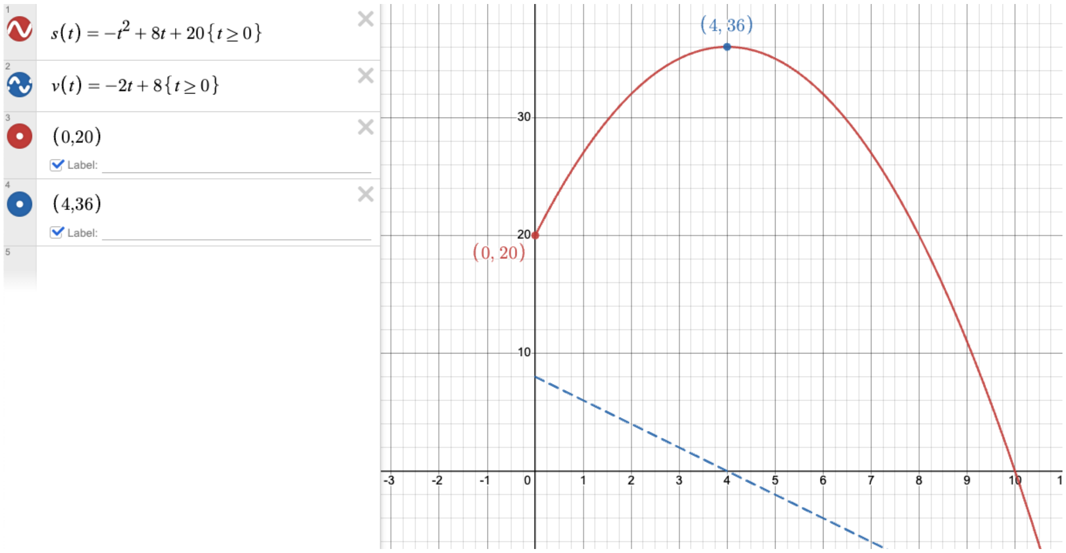
(c) Solve for the time at which the velocity of the flare is equal to 0, and then find the height of the flare at that time:

Solution:

$$v(t) = -2t + 8 = 0 \Longrightarrow t = 4$$

$$h(4) = -(4^2) + 8(4) + 20 = -16 + 32 + 20 = 36\, ft$$

(d) Shown below is the graph of $y = h(t)$. Label the points on the graph of $y = h(t)$ corresponding to the times/heights you found in parts (b) and (c). Also sketch the velocity function $y = v(t)$ on the graph:



(e) How long is the flare in the air (i.e., at what time t does the flare hit the ground)? You can answer this using the graph above, but for full credit solve for t algebraically:
(Hint: $h(t) = -t^2 + 8t + 20 = -(t^2 - 8t - 20)$; now factor in order to solve $h(t) = 0$!)

Solution: $h(t) = -t^2 + 8t + 20 = -(t^2 - 8t - 20) = -(t - 10)(t + 2)$ so $h(t) = 0$ for $t = 10$ and $t = -2$. Since we are only considering $t \geq 0$, we conclude that the flare is in the air for 10 seconds.

4. (10 points) Consider the function $f(x) = \dfrac{6x-8}{x^2+4}$

(a) Find the derivative of this function, using the Quotient Rule (you don't need to simplify your solution):

Solution: $f'(x) = \dfrac{(6)(x^2+4)-(6x-8)(2x)}{(x^2+4)^2}$

(b) Calculate $f'(0)$:

Solution: $f'(0) = \dfrac{(6)(0^2+4)-(0-8)(0)}{(0^2+4)^2} = \dfrac{24}{16} = \dfrac{3}{2}$

(c) Calculate $f(0)$:

Solution: $f(0) = \dfrac{6(0)-8}{0^2+4} = -\dfrac{8}{4} = -2$

(d) Write the equation of the tangent line for the graph $y = f(x)$ at $(0, f(0))$:

Solution:

$$y = -2 + \frac{3}{2}x$$

(e) Shown below is the graph of $y = \dfrac{6x-8}{x^2+4}$. Sketch the tangent line whose equation you found in part (c):