MAT1475, Fall 2023 **Quiz #3**
Instructor: Suman Ganguli **Wednesday, Nov 8** **Name:** ______________________

1. Suppose an object is launched straight up with an initial velocity of 50 meters per second, from an initial height of 2 meters. Then the height of the object after t seconds is given by the function:

$$h(t) = -4.9t^2 + 50t + 2$$

(a) (4 points) Find the velocity function $v(t)$, i.e., find the derivative of height function:

$$v(t) = \frac{dh}{dt} =$$

Solution:

$$v(t) = \frac{dh}{dt} = -9.8t + 50$$

(b) (3 points) What is the velocity of the object after 1 second? What is its velocity after 10 seconds?

Solution:

$$v(1) = -9.8(1) + 50 = 40.8\, m/s$$

$$v(10) = -9.8(10) + 50 = 50 - 98 = -48\, m/s$$

Note that $v(1)$ is positive because the object is still rising at that time, while $v(10)$ is negative because the object is falling.

(c) (3 points) Solve for the time when the velocity is zero (i.e., solve the equation $v(t) = 0$ for t). What does this moment in time represent, in terms of the trajectory of the object?

Solution:

$$v(t) = -9.8t + 50 = 0 \Longrightarrow t = \frac{50}{-9.8} \approx 5.1\, sec$$

This is the time when the object is momentarily still–when its trajectory switches from rising to falling. Thus, it's the time at which the object reaches maximum height.

2. Consider the functions:

$$\begin{aligned} f(x) &= x^4 \\ g(x) &= \sin x \end{aligned}$$

(a) (2 points) Write down the derivatives of these two functions:

Solution:

$f'(x) = 4x^3$

$g'(x) = \cos x$

(b) (3 points) Compute the derivative of the product $h(x) = f(x) \cdot g(x)$ using the Product Rule:

Solution:

$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) = 4x^3 \cdot \sin x + x^4 \cdot \cos x$

(c) (5 points) Write down the composite functions $f \circ g$ and $g \circ f$, and then find their derivatives using the Chain Rule:

Solution:

$(f \circ g)(x) = f(g(x)) = (\sin x)^4 = \sin^4 x$

$(g \circ f)(x) = g(f(x)) = \sin(x^4)$

$(f \circ g)'(x) = f'(g(x)) \cdot g'(x) = 4 \sin^3 x \cdot \cos x$

$(g \circ f)'(x) = g'(f(x)) \cdot f'(x) = \cos(x^4) \cdot (4x^3)$