MAT1475, Fall 2023 **Quiz #2**
Instructor: Suman Ganguli **Wednesday, October 11** **Name:** ______________________

Consider the functions:

$$\begin{aligned} f(x) &= \sqrt{x} \\ g(x) &= x^2 - 5x + 1 \end{aligned}$$

1. (4 points) Write down the derivatives of these two functions:

Solution:

$$f'(x) = \frac{1}{2\sqrt{x}}, g'(x) = 2x - 5$$

2. (8 points) Compute the derivative of the product $h(x) = f(x) \cdot g(x)$ by two different methods:

(a) First simplify $h(x) = f(x) \cdot g(x)$ by distributing the $\sqrt{x}$ term, and then compute $h'(x)$ using the Power Rule:

Solution:

$$h(x) = f(x) \cdot g(x) = \sqrt{x} \cdot (x^2 - 5x + 1) = x^2\sqrt{x} - 5x\sqrt{x} + \sqrt{x} = x^{5/2} - 5x^{3/2} + x^{1/2}$$

$$h'(x) = \frac{5}{2}x^{3/2} - \frac{15}{2}x^{1/2} + \frac{1}{2}x^{-1/2} = \frac{5}{2}x^{3/2} - \frac{15}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$$

(b) Use the Product Rule to compute $h'(x)$ directly:

Solution:

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) = \frac{1}{2\sqrt{x}} \cdot (x^2 - 5x + 1) + \sqrt{x} \cdot (2x - 5)$$

$$= \frac{1}{2}x^{3/2} - \frac{5}{2}x^{1/2} + \frac{1}{2\sqrt{x}} + 2x^{3/2} - 5\sqrt{x} = \frac{5}{2}x^{3/2} - \frac{15}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$$

3. (3 points) Write down the equation of the tangent line to the graph of $y = h(x) = f(x) \cdot g(x)$ at $x_0 = 4$:

(a) The slope of the tangent line at $x_0 = 4$ is:

Solution:

$$m_{tan} = h'(4) = \frac{5}{2} \cdot 4^{3/2} - \frac{15}{2}\sqrt{4} + \frac{1}{2\sqrt{4}} = \frac{5}{2} \cdot 2^3 - \frac{30}{2} + \frac{1}{4} = 20 - 15 + \frac{1}{4} = 5.25 = \frac{21}{4}$$

(b) The y-coordinate of the point on the graph of $y = h(x)$ for $x_0 = 4$ is:

Solution:

$$y_0 = h(4) = f(4) \cdot g(4) = \sqrt{4} \cdot (4^2 - 20 + 1) = 2 \cdot (-3) = -6$$

(c) Hence, the equation of the tangent line (in point-slope form) is:

Solution:

$$y = -6 + \frac{21}{5}(x - 4)$$