1. (10 points) Solve the following indefinite integral using the method of partial fractions:

$$\int \frac{4}{x^2 + 2x - 15}\, dx$$

Hint: start by factoring the quadratic polynomial in the denominator of the integrand, and then set up a partial fractions decomposition of the integrand in terms of those factors.

Solution: Since $x^2 + 2x - 15 = (x+5)(x-3)$, a partial fractions decomposition of the integrand takes the form:

$$\frac{4}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

We solve for the unknowns A, B by first multiplying through both sides of the equation by $(x+5)(x-3)$ in order to "clear the fraction":

$$4 = A(x-3) + B(x+5)$$

We then use "strategic substitution" to find A and B, i.e., we substitute the values of x which "zero out" one of the linear factors on the right-hand side.
Substituting $x = 3$:

$$4 = B(3+5) \Longrightarrow B = \frac{4}{8} = \frac{1}{2}$$

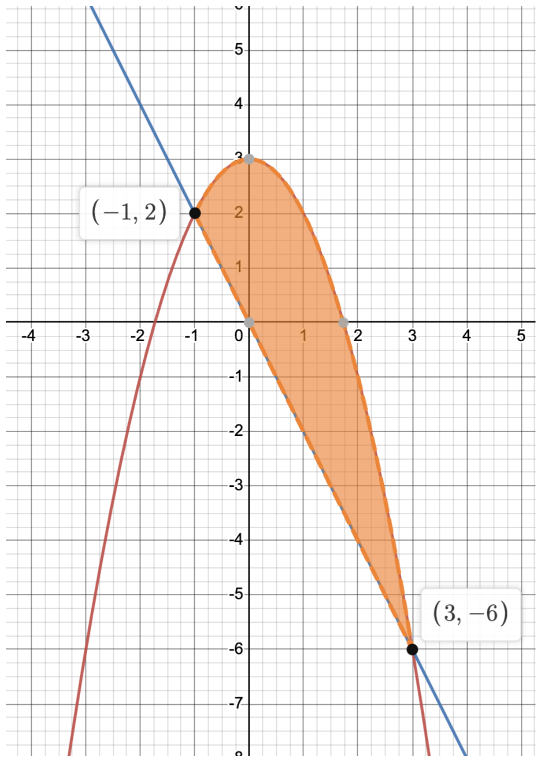
Substituting $x = -5$:

$$4 = A(-5-3) \Longrightarrow A = \frac{4}{-8} = -\frac{1}{2}$$

Hence:

$$\int \frac{4}{(x+5)(x-3)}\, dx = \int \left(\frac{-1/2}{x+5} + \frac{1/2}{x-3} \right) dx = -\frac{1}{2}\ln|x+5| + \frac{1}{2}\ln|x-3| + C$$

2. (10 points) Shown below is the graph of $y = 3 - x^2$.



(a) Add the graph of the line $y = -2x$ above, and shade in the area enclosed by the two graphs.

(b) Algebraically solve for the two values of x where the graphs intersect:

Solution: From looking at the graph above, it appears that the two graphs intersect at $x = -1$ and $x = 3$. We can confirm this by algebraically solving for where the two functions are equal:

$$3 - x^2 = -2x \Longrightarrow x^2 - 2x - 3 = 0 \Longrightarrow x^2 - 2x - 3 = 0 \Longrightarrow (x-3)(x+1) = 0 \Longrightarrow x = 3, x = -1$$

(c) Set up and solve the definite integral to find the area enclosed between the two graphs:

Solution:

$$\int_{-1}^{3} (3 - x^2) - (-2x)\, dx = \int_{-1}^{3} (-x^2 + 2x + 3)\, dx =$$

$$\left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^{3} = \left(-\frac{3^3}{3} + 3^2 + 3(3)\right) - \left(-\frac{(-1)^3}{3} + (-1)^2 + 3(-1)\right) =$$

$$\left(-\frac{27}{3} + 9 + 9\right) - \left(\frac{1}{3} + 1 - 3\right) = 11 - \frac{1}{3} = \frac{32}{3}$$