MAT1575 - Office hrs

Wed, Dec 13

Sec 5.3

Example 5.13(a) — relevant for Take-home #2!

$$\sum_{n=1}^{\infty} \frac{n}{3n-1}$$

Note that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{3n-1} = \boxed{\lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}(3-\cancel{\tfrac{1}{n}}^{\,0})}} = \frac{1}{3}$$

(b/c $\frac{1}{n} \to 0$ as $n \to \infty$)

Therefore, by the Divergence Test,
the series diverges.

WW: "Alt Series - #1"

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$$

- this alt. series converges by the alt. series test:
  - $\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}} > \ldots$
  - $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

    (b/c $\sqrt{n} \to \infty$ as $n \to \infty$)
- but the series is not absolutely convergent;
  b/c $\sum \frac{1}{n^{1/2}}$ diverges (as a p-series w/ $p = \frac{1}{2} < 1$)

Thus, the given series is conditionally convergent

WW - Alt. Series #2

$\to \frac{2}{8} = \frac{1}{4}$ as $n \to \infty$

$$\sum_{n=1}^{\infty} (-1)^n \frac{2n}{8n+3} \text{ diverges}$$

by the (Divergence Test), since

$$\lim_{n\to\infty} \frac{(-1)^n\, 2n}{8n+3} \neq 0$$

$$\sum (-1)^n \frac{2n}{8n+3}$$

$\approx$

$-\frac{1}{4} + \frac{1}{4} - \frac{1}{4} + \frac{1}{4} - \dots$

#3

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+5} \text{ is (conditionally) convergent:}$$

- not absolutely convergent, since

$$\sum \frac{\sqrt{n}}{n+5} \text{ diverges by limit-comparison}$$

with $\sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$

(diverges by p-series Test)

$\frac{\sqrt{n}}{n+5} \approx \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$ as $n \to \infty$

- convergent by the AST:
  - $$\lim_{n\to\infty} \frac{\sqrt{n}}{n+5} = \lim_{n\to\infty} \frac{\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}\left(\sqrt{n} + \frac{5}{\sqrt{n}}\right)}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n} + \frac{5}{\sqrt{n}}} = 0$$

FER #9(d)

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 - n - 1}{2n^2 + n + 1} \text{ is divergent}$$

by the Divergence Test: $\lim_{n\to\infty} a_n \neq 0$

WW - Alt Series - #5.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n^{1.1}}$$

$\frac{1}{5}\sum \frac{1}{n^{1.1}}$ convergent p-series

$p = 1.1 > 1$

is absolutely convergent b/c

WW - Alt Series #6(c)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n-1}}{n^2-6}$$

$n \to \infty$

$$\left[\frac{\sqrt{n-1}}{n^2-6} \approx \frac{\sqrt{n}}{n^2} = \frac{n^{1/2}}{n^2} = \frac{1}{n^{2-\frac{1}{2}}} = \frac{1}{n^{3/2}}\right]$$

absolutely convergent

(by limit-comparison w/ $\sum \frac{1}{n^{3/2}}$,

applied to $\sum \frac{\sqrt{n-1}}{n^2-6}$)

FER #8(c)

$$\sum_{n=1}^{\infty} \frac{5n}{10^n}$$

To apply the Ratio Test:

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{\left(5(n+1)/10^{n+1} \right)}{5n/10^n}$$

$$= \lim_{n\to\infty} \frac{\cancel{5}(n+1)}{10^{n+1}} \cdot \frac{10^n}{\cancel{5}n} = \lim_{n\to\infty} \frac{1}{10} \cdot \frac{n+1}{n} \to 1$$

$$= \frac{1}{10} < 1$$

~~Thus~~
Since $\rho = \frac{1}{10} < 1$, this series converges.