Mathematics 1575/D530, Spring 2023 **Exam #1**
Instructor: Suman Ganguli **October 11, 2023** **Name:** ____________________

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

Show all your work and simplify your answers.

1. (10 points) Evaluate the following integrals using the basic antiderivatives:

(a)

$$\int \left(3x^6 - \frac{x^3}{2} + \frac{1}{x} + \frac{1}{x^2}\right)\, dx =$$

Solution: $\dfrac{3x^7}{7} - \dfrac{x^4}{8} + \ln|x| - x^{-1} + C$

(b)

$$\int_1^4 (1 + 3\sqrt{t})\, dt =$$

Solution: $\displaystyle\int_1^4 (1 + 3\sqrt{t})\, dt = \left[t + 2t^{3/2}\right]_1^4 = (4 + 2\cdot 4^{3/2}) - (1 + 2\cdot 1^{3/2}) = (4+16) - (1+2) = 20 - 3 = 17$

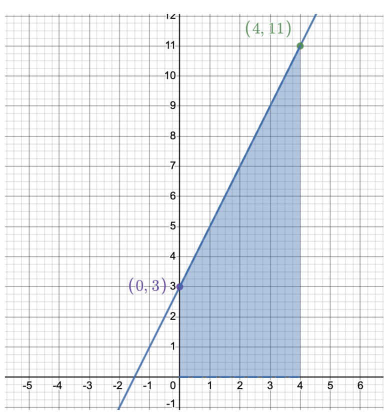
2. (10 points) Evaluate the definite integral $\displaystyle\int_0^4 (2x+3)\, dx$ in two different ways:

(a) using the Fundamental Theorem of Calculus, i.e., using an antiderivative:

Solution:

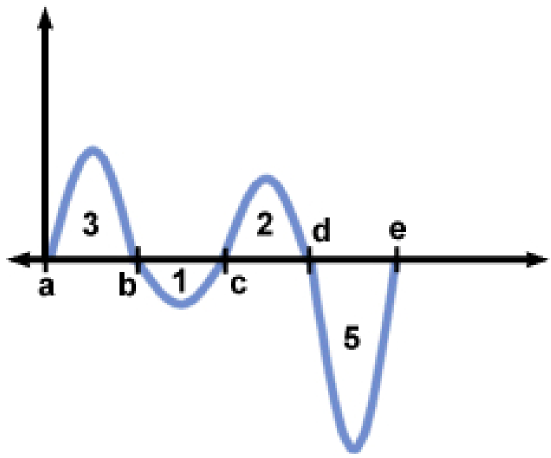
$$\int_0^4 (2x+3)\, dx = \left[x^2 + 3x\right]_0^4 = (4^2 + 3(4)) - (0^2 + 3(0)) = 16 + 12 = 28$$

(b) Use the given graph of $y = 2x + 3$ and area calculations (as on the quizzes, shade in the region of the graph corresponding to the definite integral, and label the graph with the relevant dimensions and areas):



Solution: $\displaystyle\int_0^4 (2x+3)\, dx = (4)(3) + \frac{1}{2}(4)(8) = 12 + 16 = 28$

3. (10 points) This graph shows the areas of each of the enclosed regions between the curve $y = f(x)$ and the x-axis. Use the graph to find the values of the definite integrals below:



(a) $\int_a^c f(x)\, dx =$

Solution: $3 - 1 = 2$

(b) $\int_b^c f(x)\, dx =$

Solution: -1

(c) $\int_a^e f(x)\, dx =$

Solution: $3 - 1 + 2 - 5 = -1$

(d) $\int_a^e |f(x)|\, dx =$

Solution: $3 + 1 + 2 + 5 = 11$

4. (10 points) Evaluate the following integrals using substitution:

(a) $\int e^{\cos x} \sin x\, dx =$

Solution: Substituting $u = \cos x, du = -\sin x\, dx$ yields:

$$\int e^{\cos x} \sin x\, dx = -\int e^u du = -e^{\cos x} + C$$

(b) $\int_0^1 \frac{x}{\sqrt{x^2+9}}\, dx =$

Solution: Substituting $u = x^2 + 9, du = 2x\, dx$ yields:

$$\int_0^1 \frac{x}{\sqrt{x^2+9}}\, dx = \frac{1}{2}\int_{x=0}^{x=1} \frac{du}{\sqrt{u}}\, du = \frac{1}{2}\left[2u^{1/2}\right]_{x=0}^{x=1} = \left[\sqrt{x^2+9}\right]_{x=0}^{x=1} = \sqrt{1+9} - \sqrt{0+9} = \sqrt{10} - 3$$

5. (10 points) Recall that the integration by parts formula is:

$$\int u\,dv = uv - \int v\,du \quad \text{or} \quad \int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

Use integration by parts to solve the following integrals. The appropriate choices of u and dv are given. Start by finding du and v, and then apply the integration by parts formula.

(a)

$$\int x\sin(2x)\;dx =$$

Solution: Integration by parts works with the following choices:

$$u = x \Longrightarrow du = dx$$

$$dv = \sin(2x)\,dx \Longrightarrow v = -\frac{1}{2}\cos(2x)$$

Then:

$$\int x\sin(2x)\;dx = -\frac{1}{2}x\cos(2x) - \int -\frac{1}{2}\cos(2x)\;dx = -\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\;dx$$

$$= -\frac{1}{2}x\cos(2x) + \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$$

(b)

$$\int \frac{\ln x}{x^3}\;dx =$$

Solution: Integration by parts works with the following choices:

$$u = \ln x \Longrightarrow du = \frac{1}{x}dx$$

$$dv = \frac{1}{x^3}dx \Longrightarrow v = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$$

Then:

$$\int \frac{\ln x}{x^3}\;dx = (\ln x)\left(-\frac{1}{2x^2}\right) - \int \left(-\frac{1}{2x^2}\right)\frac{1}{x}\;dx = -\frac{\ln x}{2x^2} + \frac{1}{2}\int \frac{1}{x^3}\;dx$$

$$= -\frac{\ln x}{2x^2} + \frac{1}{2}\left(-\frac{1}{2x^2}\right) + C = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C$$