MAT1575, Fall 2023 **Quiz #2**
Instructor: Suman Ganguli **Due: Wed, Oct 4** **Name:** ____________________

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1. (5 points) Evaluate the following indefinite integrals (i.e., find the general antiderivatives of the given functions):

(a)

$$\int \left(12x^2 - 8x + \frac{1}{x} + \frac{3}{\sqrt{x}}\right) dx =$$

Solution: Since $\frac{1}{\sqrt{x}} = x^{-1/2}$, we know that $\int \frac{1}{\sqrt{x}}\, dx = 2x^{1/2} + C = 2\sqrt{x} + C$

Hence:

$$\int \left(12x^2 - 8x + \frac{1}{x} + \frac{3}{\sqrt{x}}\right) dx = \frac{12x^3}{3} - \frac{8x^2}{2} + \ln x + 6\sqrt{x} + C = 4x^3 - 4x^2 + \ln x + 6\sqrt{x} + C$$

Note that we can check that we have found the correct antiderivative by differentiating the result:

$$\frac{d}{dx}\left(4x^3 - 4x^2 + \ln x + 6\sqrt{x} + C\right) = 4(3x^2) - 4(2x) + \frac{1}{x} + 6\left(\frac{1}{2\sqrt{x}}\right) = 12x^2 - 8x + \frac{1}{x} + \frac{3}{\sqrt{x}}$$

(b)

$$\int \left(\frac{e^x}{2} + \sin x\right) dx =$$

Solution:

$$\int \left(\frac{e^x}{2} + \sin x\right) dx = \frac{e^x}{2} - \cos x + C$$

2. (5 points) Use substitution to solve the following integrals:

(a)

$$\int \frac{1}{7x + 2}\, dx =$$

Solution: $u = 7x + 2 \implies du = 7\, dx \implies dx = \frac{1}{7}\, du$

$$\int \frac{1}{7x+2}\, dx = \frac{1}{7}\int \frac{1}{u}\, du = \frac{1}{7}\ln u + C = \frac{1}{7}\ln(7x+2) + C$$

(b)

$$\int t^2 \cos(1 + t^3)\, dt =$$

Solution: $u = 1 + t^3 \implies du = 3t^2\, dt \implies t^2\, dt = \frac{1}{3}\, dt$

$$\int x^2 \cos(1 + x^3)\, dx = \frac{1}{3}\int \cos(u)\, du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin(1 + t^3) + C$$

3. (5 points) (a) Evaluate the following definite integral:

$$\int_{-3}^{3} \left(x^2 + 4\right)\, dx =$$

Solution:

$$\int_{-3}^{3} \left(x^2 + 4\right)\, dx = \left[\frac{x^3}{3} + 4x\right]_{-3}^{3} \left[\frac{3^3}{3} + 4(3)\right] - \left[\frac{(-3)^3}{3} + 4(-3)\right] = (9+12) - (-9-12) = 21 + 21 = 42$$

(b) Sketch the graph of $y = x^2 + 4$ and shade in the region on the graph corresponding the definite integral in (a):