MAT1575, Spring 2023 **Homework - Integration by Substitution**
Instructor: Suman Ganguli **Due: Wednesday, Oct 4** **Name:** ____________________

For each of the following:

- write down "an appropriate" substitution u (for some of the exercises, u is given–use those exercises to understand why that choice of u works)
- find du by differentiating u
- make the substitution into the given integral to transform it into an integral in the new variable u
- find the general antiderivative with respect to u
- resubstitute to to get the antiderivative to the original integral in the original variable

1. $\int (x-7)^3\,dx =$

Solution: $u = x - 7 \Longrightarrow du = dx$

$$\int (x-7)^3\,dx = \int u^3\,dx = \frac{u^4}{4} + C = \frac{(x-7)^4}{4} + C$$

2. $\int \cos(\theta + \pi)\,d\theta =$

Solution: $u = \theta + \pi \Longrightarrow du = d\theta$

$$\int \cos(\theta+\pi)\,d\theta = \int \cos u\,du = \sin u + C = \sin(\theta+\pi) + C$$

3. $\int 2t\sqrt{t^2+1}\,dt =$

Solution: $u = t^2 + 1 \Longrightarrow du = 2t\,dt$

$$\int 2t\sqrt{t^2+1}\,dt = \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(t^2+1)^{3/2} + C$$

4. $\int \frac{(\ln x)^2}{x}\,dx =$

Solution: $u = \ln x \Longrightarrow du = \frac{1}{x}\,dx$

$$\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \frac{u^3}{3} + C = \frac{(\ln x)^3}{3} + C$$

5. $\int \sin^2\theta\cos\theta\,d\theta =$

Solution: $u = \sin\theta \Longrightarrow du = \cos\theta\,d\theta$

$$\int \sin^2\theta\cos\theta\,d\theta = \int u^2\,du = \frac{u^3}{3} + C = \frac{\sin^3\theta}{3} + C$$

6. $\int (4x+5)^9 \, dx$

Solution: $u = 4x+5 \Longrightarrow du = 4\,dx \Longrightarrow dx = \frac{1}{4}\,du$

$$\int (4x+5)^9 \, dx = \frac{1}{4}\int u^9 \, du = \frac{1}{4}\frac{u^{10}}{10} + C = \frac{(4x+5)^{10}}{40} + C$$

7. $\int \cos(5x) \, dx$

Solution: $u = 5x \Longrightarrow du = 5\,dx \Longrightarrow dx = \frac{1}{5}\,du$

$$\int \cos(5x) \, dx = \frac{1}{5}\int \cos(u) \, du = \frac{1}{5}\sin(u) + C = \frac{1}{5}\sin(5x) + C$$

8. $\int xe^{x^2} \, dx$

Solution: $u = x^2 \Longrightarrow du = 2x\,dx \Longrightarrow x\,dx = \frac{1}{2}\,du$

$$\int xe^{x^2} \, dx = \frac{1}{2}\int e^u \, du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

9. $\int \frac{dz}{(5-2z)^2}$

Solution: $u = 5-2z \Longrightarrow du = -2\,dz \Longrightarrow dz = -\frac{1}{2}\,du$

$$\int \frac{dz}{(5-2z)^2} = -\frac{1}{2}\int \frac{du}{u^2} = -\frac{1}{2}\int u^{-2} \, du = -\frac{1}{2}\frac{u^{-1}}{-1} + C = \frac{1}{2}(5-2z)^{-1} + C = \frac{1}{2(5-2z)} + C$$

10. $\int \frac{x^2}{x^3+1} \, dx$

Solution: $u = x^3+1 \Longrightarrow du = 3x^2\,dx \Longrightarrow x^2\,dx = \frac{1}{3}\,du$

$$\int \frac{x^2}{x^3+1} \, dx = \frac{1}{3}\int \frac{1}{u} \, du = \frac{1}{3}\ln(u) + C = \frac{1}{3}\ln(x^3+1) + C$$