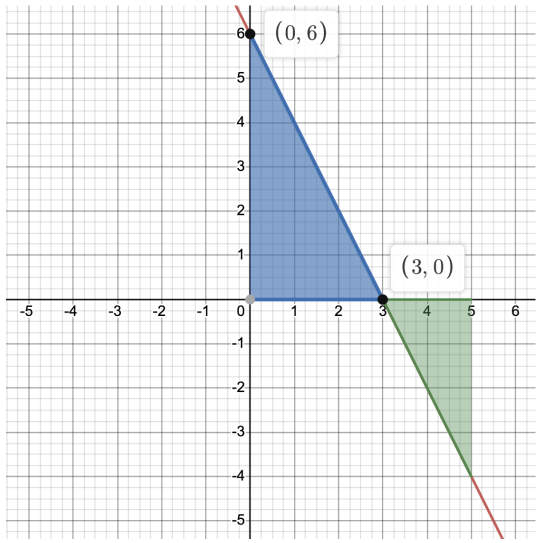
Mathematics 1575, Spring 2023 **Quiz #1**
Instructor: Suman Ganguli **Due: Wednesday, Sept 20** **Name:** ______________________

Consider the linear function: $f(x) = 6 - 2x$

1. (3 points) Sketch the graph $y = f(x)$. Label the x- and y-intercepts with their coordinates.



2. (a) (2 points) Shade in the two triangles(s) on the graph corresponding the definite integral $\int_0^5 (6-2x)\,dx$ and label each triangle with its base, height, and area.

Solution: The triangle on the left, above the x-axis over the interval $[0, 3]$, clearly has height $h = 6$ and base $b = 3$, and hence its area is $A_1 = \frac{1}{2}(3)(6) = 9$. The triangle on the right, below the x-axis and along the interval $[3, 5]$, has height $h = 4$ and base $b = 2$, and so its area is $A_2 = \frac{1}{2}(2)(4) = 4$.

(b) (2 points) Compute the value of the definite integral in terms of those areas:

Solution: The definite integral corresponds to the "net area" between $y = f(x)$ and the x-axis, i.e., the area of the right triangle A_2 counts negatively for the definite integral since it is below the x-axis. Hence:

$$\int_0^5 (6-2x)dx = A_1 - A_2 = 9 - 4 = 5$$

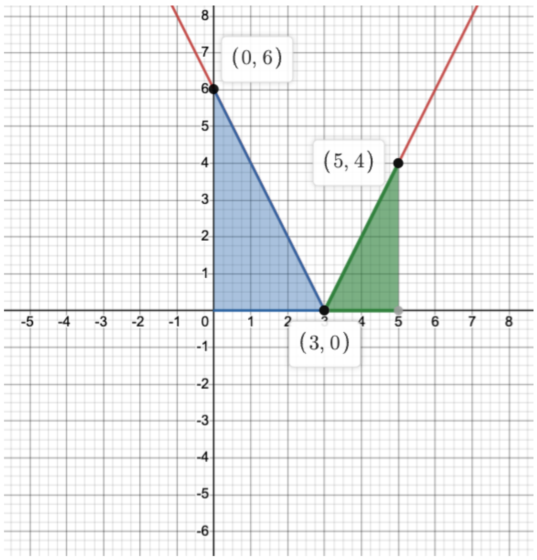
3. (3 points) Now evaluate the same definite integral using the Fundamental Theorem of Calculus (i.e., by finding and evaluating an antiderivative of $f(x) = 6 - 2x$):

Solution:

$$\int_0^5 (6-2x)\,dx = [6x - x^2]_0^5 = [6(5) - 5^2] - [6(0) - 0^2] = (30 - 25) - (0 - 0) = 5$$

(Extra credit!) What is the value of $\int_0^5 |6 - 2x|\,dx$? Explain how you arrive at your answer. You can also sketch the corresponding graph.

Solution: Recall that the absolute value just reflects the portions of the graph that are below the x-axis to above the x-axis:



Hence, the definite integral $\int_0^5 |6-2x|\,dx$ corresponds to the sum of the areas of the same two triangles as in #2, but area of the triangle on the right, over the interval $[3,5]$, now counts positively since it is above the x-axis.

$$\int_0^5 |6-2x|\,dx = 9+4 = 13$$