MAT2440/D648 **Quiz #2**
Instructor: Suman Ganguli **Wednesday, March 30** **Name:** ______________________

Question:	1	2	Total
Points:	5	10	15
Score:			

Recall the formal definitions of even and odd integers:

Definition: An integer n is *even* if $n = 2k$ for some integer k. An integer n is *odd* if $n = 2k + 1$ for some integer k.

1. (5 points) Prove that the sum of two odd integers is even.
(Hint: use a direct proof to prove "If m and n are both odd integers, then $m + n$ is an even integer.")

Proof:.

Solution:

(For a direct proof, we assume the antecedent–in this case that m and n are odd integers–and proceed to prove the consequence–that that implies $m * n$ is also odd.)

Take any two odd integers m and n. By definition of being odd, $m = 2k + 1$ and $n = 2j + 1$ for integers j, k. (Note that since m and n may be different odd integers, we must assume two different representations of those odd integers, $2k + 1$ and $2j + 1$, respectively.)

Then $m + n = (2k + 1) + (2j + 1) = 2k + 2j + 2 = 2(k + j + 1)$, where $k + j + 1$ is clearly an integer. Thus, $m + n$ fulfills the definition of being odd.

2. (10 points) Let's prove of the statement: "If $m * n$ is even, then either m is even or n is even."

a. We will provide a proof by contraposition. The statement is a conditional $p \longrightarrow q$, where the propositions are

$p =$ ______________________

Solution: $p =$ "$m * n$ is even"

$q =$ "m is even or n is even"

b. For a proof by contraposition, we will prove the logically equivalent contrapositive $\neg q \rightarrow \neg p$. Write down $\neg p$, and simplify $\neg q$ using DeMorgan's Law (and using the fact that "m is not even" is logically equivalent to "m is odd"):

$\neg p =$ ______________________

$\neg q = \neg$ (m is even or n is even) $=$ ______________________

Solution: $\neg p =$ "$m * n$ is not even" $=$ "$m * n$ is odd"
Applying DeMorgan's Law to formulate $\neg q$, we have:

$$\neg q \equiv \neg(m \text{ is even or } n \text{ is even}) \equiv \neg(m \text{ is even}) \text{ and } \neg(n \text{ is even}) \equiv (m \text{ is odd}) \text{ and } (n \text{ is odd})$$

c. Now write out a direct proof of $\neg q \rightarrow \neg p$:

Proof:.

Solution: In general, for a proof by contraposition of a statement "if p, then q," we prove the logically equivalent contraposition "if $\neg q$, then $\neg p$."

Thus, for the statement "If $m * n$ is even, then either m is even or n is even", a proof by contraposition begins by assuming $\neg q$, i.e., it is *not* the case that either m is even or n is even, which we have shown is equivalent to assuming m is odd *and* n is odd. Then we must use that assumption to show $\neg p$, i.e., that $m * n$ is odd.

So the proof by contraposition is as follows (it's sufficient to just write this!):

For a proof by contraposition, assume m and n are both odd integers. By definition, $m = 2k+1$ and $n = 2j+1$ for integers j, k. Then

$$m * n = (2k+1)(2j+1) = 4jk + 2j + 2k + 1 = 2(2jk + j + k) + 1$$

where $2jk + j + k$ is clearly an integer. Thus, $m * n$ is odd. This proves the theorem.