MAT2440/D648 **Exam #1 - take-home component**
Instructor: Suman Ganguli **Due: Monday, March 14** **Name:** ______________________

Question:	1	2	3	4	5	Total
Points:	10	15	5	15	15	60
Score:						

Please **work on this exam individually**, but study the textbook/slides and similar examples and exercises. If possible, print this out this pdf and write your solutions in the spaces provided below. Otherwise, write your solutions on blank pieces of paper (in which case, you don't need to rewrite the statements of the questions, but write your solutions in order, and number them. Submit your written solutions in class on Monday. We will have an in-class component to the exam on Monday.

1. Recall that a compound proposition ϕ is *satisfiable* if there is an assignment of truth values to its variables that makes the proposition true, and *unsatisfiable* if every assignment of truth values to its variables makes the proposition false.

   Determine whether the following compound propositions are satisfiable or unsatisfiable by constructing their truth tables. If the proposition is satisfiable, circle the line(s) of the truth table that demonstrates that. If the proposition is unsatisfiable, explain what feature of the truth table demonstrates that.

   a. (5 points)

   $(p \longleftrightarrow q) \land \neg p$ is **satisfiable/unsatisfiable**:

p	q	$p \longleftrightarrow q$	$\neg p$	$(p \longleftrightarrow q) \land \neg p$
T	T			
T	F			
F	T			
F	F			

   b. (5 points)

   $p \land (p \longrightarrow q) \land \neg q$ is **satisfiable/unsatisfiable**::

p	q	$p \longrightarrow q$	$p \land (p \longrightarrow q)$	$\neg q$	$p \land (p \longrightarrow q) \land \neg q$
T	T				
T	F				
F	T				
F	F				

2. Consider the conditional statement $[(p \vee q) \wedge \neg p] \longrightarrow q$.

a. (6 points) Construct the truth table for $[(p \vee q) \wedge \neg p] \longrightarrow q$:

p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$	$[(p \vee q) \wedge \neg p] \longrightarrow q$
T	T				
T	F				
F	T				
F	F				

b. (3 points) What feature of its truth table establishes that $[(p \vee q) \wedge \neg p] \longrightarrow q$ is a tautology? (In other words, what is the definition of a tautology, in terms of truth tables?)

c. (3 points) Recall that each rule of inference we discussed is based on a certain tautology. The rule of inference associated with $[(p \vee q) \wedge \neg p] \longrightarrow q$ is called *disjunctive syllogism*:

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Briefly describe the relationship between the tautology $[(p \vee q) \wedge \neg p] \longrightarrow q$ and this rule of inference. Here are some terms you could use in your answer:

- the propositions above the horizontal line in a rule of inference, in this case $p \vee q$ and $\neg p$, are called the *hypotheses*
- the proposition below the horizontal line, q, is called the *conclusion* of the rule of inference
- in the conditional statement $[(p \vee q) \wedge \neg p] \longrightarrow q$, the part before the arrow, $(p \vee q) \wedge \neg p$, is called the *antecedent*, and q is called the *consequence*.)

d. (3 points) Consider the following premises: "I will choose soup or salad" and "I will not choose soup." Apply the rule of inference above to these premises in order to reach a conclusion:

$$\begin{array}{r} p \vee q = \\ \neg p = \\ \hline \therefore q = \end{array}$$

3. (5 points) Write a a proof of the following theorem:

If a compound proposition ϕ is unsatisfiable, then $\neg\phi$ is a tautology.

(Hint: give a "direct proof" by assuming a given compound proposition ϕ is unsatisfiable, and use the definition of unsatisfiability to describe the truth table of ϕ. Show how this implies $\neg\phi$ fulfills the definition of a tautology. It may also be helpful to look at the truth tables of a tautology and an unsatisfiable proposition, such as in #1 and #2.)

4. Shown below is the truth table for the “exclusive-or” logical connective $\oplus$:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

a. (4 points) What is the difference between the “exclusive-or” logical connective $\oplus$ and the “inclusive-or” logical connective $\vee$? Why do you think $\oplus$ called “exclusive” and $\vee$ called “inclusive”? Give a brief explanation in terms of the truth tables of the two logical connectives.

b. (3 points) Many restaurant menus contain a statement such as “Every entree comes with a choice of soup or salad.” Do you think the correct interpretation of the “or” in such a statement is exclusive-or or inclusive-or? Justify your answer.

c. (3 points) Among the prerequisites for MAT2440 is “CST1201 or CST2403 or MAT1630.” Do you think the correct interpretation of “or” in this statement is exclusive-or or inclusive-or? Justify your answer.

d. (5 points) Show that $p \oplus q$ is logically equivalent to $(p \vee q) \wedge \neg(p \wedge q)$ by constructing the truth table for $(p \vee q) \wedge \neg(p \wedge q)$, and comparing it to the truth table for $p \oplus q$:

5. Consider the two-place predicate

$$L(x, y) = \text{“}x \text{ lives in borough } y\text{”}$$

Let the domain for x consist of all CityTech students, and the domain for y consist of the five boroughs of NYC, i.e., the set {Manhattan, Brooklyn, Queens, Bronx, Staten Island}.

a. (9 points) Translate the following statements of predicate logic into English sentences. For full credit, write a "natural" English translation, i.e., a sentence one would say in normal conversation. (A "literal" translation will receive partial credit.)

- $\neg\exists x L(x, \text{Staten Island})$

- $\forall x \exists y L(x, y)$

- $\forall y \exists x L(x, y)$

b. (6 points) Express each of the following sentence as a statement of predicate logic using quantifiers, logical connectives and the predicate $L(x, y)$:

- "All CityTech students live in Brooklyn, Queens, or Manhattan."

- "There are CityTech students who don't live in New York City (i.e., who don't live in any of the five boroughs)."

c. (extra credit) Suppose you have access to a CityTech database which contains the home addresses of all CityTech students. Describe an algorithm that would tell you whether $\neg\exists x L(x, \text{Staten Island})$ is true or false.