MAT 2580 **Review Exam3** **Prof. Ghosh-Dastidar**

1. Evaluate each determinant. Show complete work.

   (a) $\begin{vmatrix} 3 & -2 & 1 \\ 3 & -1 & -2 \\ 3 & -2 & -3 \end{vmatrix}$

   (b) $\begin{vmatrix} 3 & 4 & 5 \\ -4 & 6 & 3 \\ 1 & -4 & 3 \end{vmatrix}$

   (c) $\begin{vmatrix} 0 & a & b \\ 0 & c & d \\ 0 & x & y \end{vmatrix}$

2. Use row operations to show that the determinants are all zero.

   (a) $\begin{vmatrix} 12 & 13 & 14 \\ 15 & 16 & 17 \\ 18 & 19 & 20 \end{vmatrix}$

   (b) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix}$

3. What value of x makes the determinant -4?

   $\begin{vmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{vmatrix}$

4. Find whether the following matrix is invertible or not. Singular or non-singular? **Do not find the inverse.** Show your work.

   $\begin{vmatrix} 2 & 4 & 1 & 2 \\ -3 & 0 & 0 & 0 \\ 1 & 3 & 2 & 1 \\ -2 & 9 & 3 & 1 \end{vmatrix}$

5. Let A and B be 4 x 4 matrices with det(A) = -1 and det(B)=2. Then compute

(a) det(AB)

(b) $\det(B^5)$

(c) det(2A)

(d) $\det(A^T A)$

(e) $\det(B^{-1}AB)$

(f) $\det(AB)^T$

6. True or False. Explain your answer mathematically.
   (a) If A is invertible and 1 is an eigenvalue for A, then 1 is an eigenvalue for A^{-1}.

   (b) Each eigenvalue of A is also an eigenvalue of A^2.

   (c) Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.

7. Find the eigenvalues of the following matrices.

   (a) $\begin{bmatrix} 3 & -2 & 8 \\ 0 & 5 & -2 \\ 0 & -4 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 5 & -6 & -7 \\ 2 & 4 & 5 & 2 \\ 0 & 0 & -7 & -4 \\ 0 & 0 & 3 & 1 \end{bmatrix}$

8. Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$. Show that the characteristic polynomial of A is

$$\lambda^2 - (\text{trace } A)\lambda + \det(A).$$

9, Construct bases for the column space and the null space of the given matrix. Justify your answer.

$$A = \begin{bmatrix} 5 & 3 & 2 & -6 & -8 \\ 4 & 1 & 3 & -8 & -7 \\ 5 & 1 & 4 & 5 & 19 \\ -7 & -5 & -2 & 8 & 5 \end{bmatrix}$$