

# Class #20 - Fri Oct. 15

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Last time?

→ i.e.,  $i^2 = -1$

introduced  $i = \sqrt{-1}$  ("imaginary unit")

↑  
# 1

⇒ "imaginary #s"  
 $b \cdot i$  (where  $b$  is any real #)

Ex:  $7i, i\sqrt{2}, \frac{i}{4}$

→ square root of any negative # is an imaginary #

Ex:  $\sqrt{-2} = \sqrt{(-1)(2)} = \sqrt{-1}\sqrt{2} = i\sqrt{2}$

# Arithmetic w/ imaginary #s

(use  $i^2 = -1$ )

(2)

Examples :

$$(1) \quad \left(\frac{i}{4}\right)^2 = \left(\frac{i}{4}\right) \cdot \left(\frac{i}{4}\right) = \frac{i^2}{16} = \frac{-1}{16} = -\frac{1}{16}$$

$$\left(\text{and so } \sqrt{-\frac{1}{16}} = \frac{i}{4}\right)$$

$$(2) \quad i^3 = (i \cdot i) i = i^2 \cdot i = (-1) \cdot i = -i$$

$$*i \quad i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

$$*i \quad i^5 = i^4 \cdot i = (1) \cdot i = i$$

$$*i \quad i^6 = i^5 \cdot i = (i) \cdot i = -1$$

$$i^7 = i^6 \cdot i = (-1) \cdot i = -i$$

$$i^8 = i^7 \cdot i = (-i) \cdot i = -(-1) = 1$$

$$i^9 = i^8 \cdot i = 1 \cdot i = i$$

# Powers of $i$

$i^0 = 1$	$\swarrow \times i$
$i^1 = i$	$\swarrow \times i$
$i^2 = -1$	$\swarrow \times i$
$i^3 = -i$	$\swarrow \times i$
$i^4 = 1$	$\swarrow \times i$
$i^5 = i$	$\swarrow \times i$
$i^6 = -1$	$\swarrow \times i$
$i^7 = -i$	$\swarrow \times i$
$i^8 = 1$	

"cyclic pattern"  
 "the powers of  $i$  form a cycle of length 4"

$\therefore i^{64} = i \cdot i \cdot i \dots \cdot i = 1$  (b/c 64 is a multiple of 4!)  
 $i^{65} = i$   
 $i^{66} = -1$   
 $\Rightarrow i^{67} = -i$

(4)

## Complex #s

Ex:  $7 + 7i, 1 - i, 5 + i\sqrt{2}, \dots$

$\underbrace{\hspace{15em}}$   
 "a + bi" (where a, b are real #s)  
 ↑                    ↑  
 "real part"        "imaginary part"

Arithmetic w/ complex #s.

- addition/subtraction: "combine like terms"  
 i.e. add real parts and imaginary parts.

Ex:  $(9 - 4i) + (-1 - 8i)$

$$= 8 + (-4i - 8i) = \boxed{8 - 12i}$$

## Multiplication of complex #s

5

Ex:  $(7 + 6i) \cdot (-1 - 6i)$  FOIL!

$$= -7 - 42i - 6i + (6i)(-6i)$$

$$= -7 - 48i - 36i^2 \quad \left. \vphantom{-36i^2} \right\} i^2 = -1$$

$$= -7 - 48i - 36(-1)$$

$$= -7 - 48i + 36$$

$$= 29 - 48i$$

Tuesday: Division of complex #s

(use "complex conjugates")

Read  
§ 8.8