Class #40 - Tues Dec 7

Schedule :

WebWork!

this week :

today (Class #40) - finish exponentials / logarithms

Tues (#41) } introduce trigonometry

Fri (#42) }

Quiz #4 (due this Fri)

↳ discuss + post. Midterm : Exam #3

next week

→ Tues Dec 14 (Class #43) } a little more trig.

Thurs Dec 16 - " 44 } review

Fri Dec 17 - " 45 }

- post final exam

- due Mon Dec 20

due Wed Dec 15

WebWork exercises → written solutions

Today:

- "Exponential Equations"
- "Logarithmic Functions"

$\Rightarrow$ "Exponential Equations – Calc"

#1/ $4^x = 256$

(hint: express 256 as a power of 4)

$4^x = 4^{4}$

$\Rightarrow \boxed{x = 4}$

$4^{-1} = \frac{1}{4}$ ×4
$4^0 = 1$ ×4
$4^1 = 4$ ×4
$4^2 = 16$ ×4
$4^3 = 64$ ×4
$4^4 = 256$

$(4)^4 = (2^2)^4 = 2^8 = 256$

#2/ $2^{-x} = 32$

$2^{(-x)} = 2^{5}$

$\Rightarrow -x = 5 \Rightarrow \boxed{x = -5}$

Check: $2^{-x} = 2^{-(-5)} = 2^5 = 32$ ✓

Last time: simple exponential equations

(a) what is the x-value where $8^x = 2$?

$8^{(x)} = 2$

$x = 1/3$

(since $8 = 2^3$, $2 = \sqrt[3]{8} = 8^{1/3}$)

hint: x is a fraction!

(b) x-value such that $y = 8^x = \frac{1}{64}$?

$8^x = \frac{1}{64} = \frac{1}{8^2} = 8^{-2}$

hint: y is a negative integer.

$8^x = ?$

$x = -2$

Now we will look at (define) "logarithmic" functions.

(why? b/c they will allow us to solve equations such as

$8^x = 10$ } we could use a calculator...

Logarithmic functions

$y = \log_b x$ (log function with base b)

means $b^y = x$

Rewrite in "exponential form"

$\log_{10}(0.01) = -2$

$\Rightarrow 10^{-2} = 0.01 = \frac{1}{100} = \frac{1}{10^2}$

Ex: $\log_2 x$

$\log_2 16 = ?$ $\Leftrightarrow 2^? = 16$

$2^4 = 16$

$\Rightarrow \log_2 16 = 4$

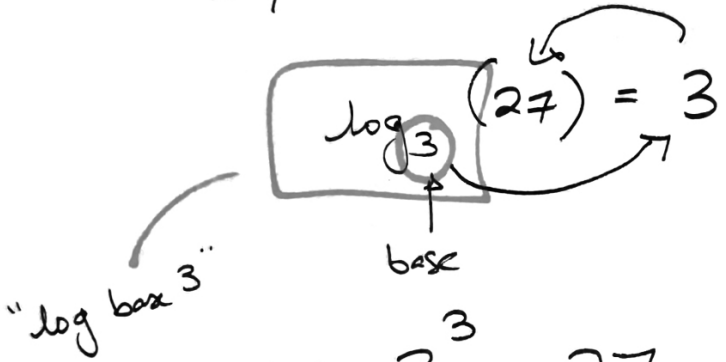
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$

$\log_3(243) = 5$

$3^5 = 243$

Review: "Logarithmic Functions"

#1/ Rewrite in terms of exponents:



$$\Longrightarrow 3^3 = 27$$

#2 / $\log_{10}(0.00001) = \boxed{-5}$

$$10^{\boxed{-5}} = 0.00001$$

#3 / $\log_b(24) = x \iff \boxed{b^x = 24}$

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* Powers of 10.

$10^0 = 1$

$10^{-1} = \frac{1}{10} = 0.1$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

$10^{-3} = \frac{1}{1000} = 0.001$

Properties of logarithms:

$$\log_b(a^x) = x \log_b(a)$$

"ln x" — "natural log" — logarithm w/ base e

$$\ln(a^x) = x[\ln a]$$

Ex: (Exp Eqn-Calc, #1)

Solve: $a^x = 6567$

Ex: $\ln_e(32)$

$= \ln(2^5) = 5(\ln 2)$

≈ 3.466

(b/c $e^{3.466} \approx 32$)

Step 1: "Take $\log_{10}$(or ln) of both sides":

$$\log_{10}(a^x) = \log_{10}(6567)$$

$$x[\log_{10}(a)] = [\log_{10}(6567)] \Rightarrow x = \frac{\log(6567)}{\log(a)}$$

Class #41 - Thurs, Dec 9

- recap schedule.
- one more exponential equation (using logarithms)
- trigonometry

$\log_b x = ?$
$\Leftrightarrow b^? = x$

- look at Quiz #4 ...

Example 10.41 (OpenStax, Sec. 10.5)

Solve: $5^x = 11$ ($1 < x < 2$; b/c $5^1 < 11 < 5^2$)

b/c $y = 5^x$ is an "increasing function"

Solve for x:

$$5^x = 11$$

Divide by 11? $\frac{5^x}{11} = 1$

Divide by 5? $\frac{5^x}{5} = \frac{11}{5} \implies 5^{x-1} = \frac{11}{5}$

→ we need to use logarithms!
That will let us "bring the variable x down"
(out of the exponent)

① "Take the logarithm" of both sides
"log" or "ln" (log₁₀ or log base e, respectively)

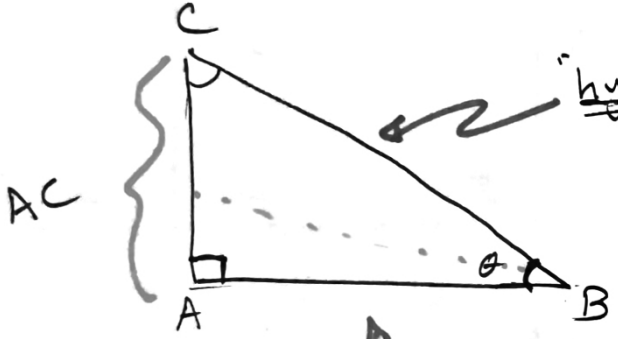
$$\ln(5^x) = \ln(11)$$

$$x \cdot (\ln 5) = \ln(11) \implies \boxed{x = \frac{\ln(11)}{\ln(5)}}$$

Trigonometry

"measuring triangles"

"Right" triangles (ie., triangle w/ a "right" (90°) angle)



"hypotenuse" = BC
(longest side - opposite the right angle)

$$\sin(B) = \frac{opp}{hyp} = \frac{AC}{BC}$$

defn of the sine of an angle

$$\cos(B) = \frac{adj}{hyp} = \frac{AB}{BC}$$

defn of cosine.

$$\sin(C) = \frac{AB}{BC}$$

$$\cos(C) = \frac{AC}{BC}$$

Class #42 - Fri, Dec 10

- schedule -

today : - finish WebWork : "Exp Functions" ← due today; "Exp Equations", "Logarithmic Functions" → ex W Tues

- hand in Quiz #4

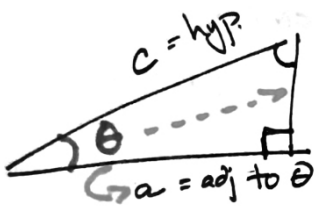
today - Wed : Exam #3 { Webwork set ("Midterm - Exam3") - hand in written solutions

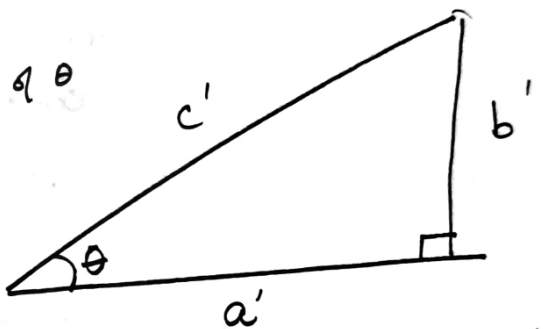
[class : next Tues / Thurs / Fri]

- next Fri (Dec 17) : post review Final Exam (WebWork exercises + written solutions)
  - due Mon Dec 20

Trigonometry

- introduced "trig ratios" on Tuesday





"similar triangles" (equal angles)

$$\sin\theta = \frac{opp}{hyp} = \frac{b}{c} = \frac{b'}{c'} \Rightarrow \text{cosecant: } \csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp}$$

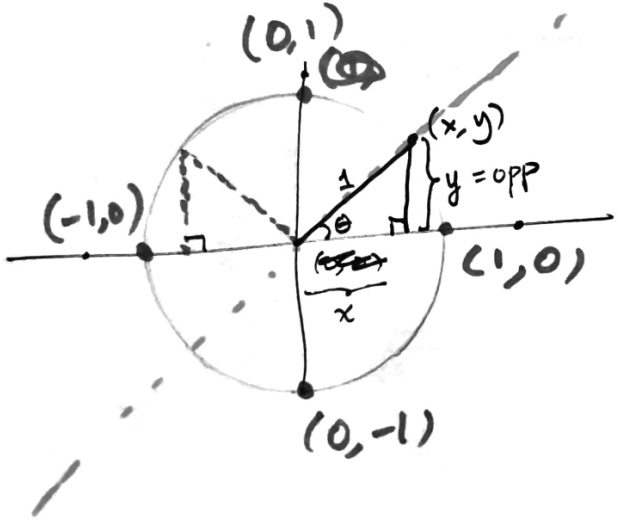
$$\cos\theta = \frac{adj}{hyp} = \frac{a}{c} = \frac{a'}{c'} \Rightarrow \sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} = \frac{b}{a} = \frac{b'}{a'} \Rightarrow \cot\theta = \frac{1}{\tan\theta} = \frac{adj}{hyp}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \left(\text{b/c } \frac{\sin\theta}{\cos\theta} = \frac{(opp/hyp)}{(adj/hyp)} = \frac{opp}{\cancel{hyp}} \cdot \frac{\cancel{hyp}}{adj} = \frac{opp}{adj}\right)$$

"Unit circle" - circle centered at (0,0) w/ radius r = 1

equation of unit circle: $x^2 + y^2 = 1$

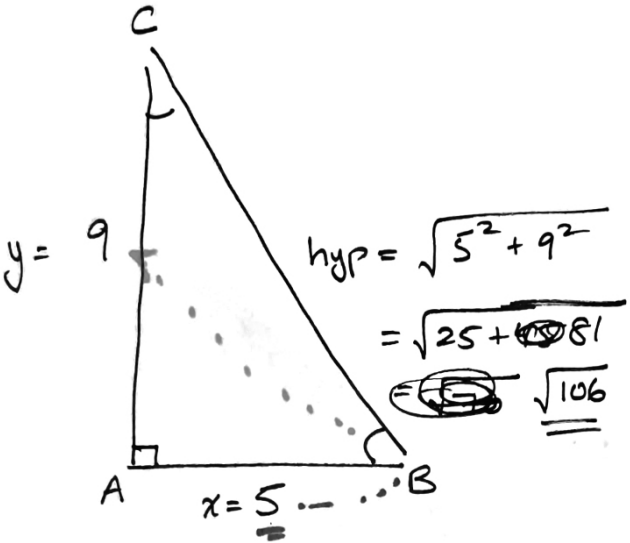


$\sin\theta = \frac{y}{1} = y$

$\cos\theta = \frac{x}{1} = x$

$\tan\theta = \frac{y}{x} \left(= \frac{\text{rise}}{\text{run}} \right)$

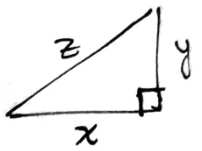
Trig Ratios #3



$hyp = \sqrt{5^2 + 9^2}$

$= \sqrt{25 + 81}$

$= \sqrt{106}$



Pythagorean Thm:

$x^2 + y^2 = z^2$

$\Rightarrow z = \sqrt{x^2 + y^2}$

$\sin B = \frac{opp}{hyp} = \frac{9}{\sqrt{106}}$

$\cos B = \frac{adj}{hyp} = \frac{5}{\sqrt{106}}$

$\tan B = \frac{9}{5}$