Class #37 - Tues Nov 30

- focus on keeping up with the WebWork!

(last Tues : complex fractions and fractional equations)

- Today (and Thursday)
  - exponentials and logarithms — "$\log_{10} x$" or "$\ln x$"
    - "Integer Exponents" (review?)
    - "Exponential Functions"

Integer exponents :

$$b^a$$

a ← "power" or "exponent"

b ← "base"

"b raised to the power a"

positive integer exponents :

$$2^3 = \underbrace{2 \times 2 \times 2}$$

$$b^a = \underbrace{b \cdot b \cdot b \cdots \cdot b}_{a \text{ times.}}$$

when "a" is a positive integer.

$$3^5 = \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{5 \text{ factors}} = \underline{243}$$

$$5^3 = 5 \times 5 \times 5 = 25 \times 5 = \underline{125}$$

"Powers of 3" (will lead us to negative exponents)

x	3^x ("exponential function")
-2	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3}$
0	$\boxed{3^0 = 1}$
1	$3^1 = 3$
2	$3^2 = 3 \cdot 3 = 9$
3	$3^3 = 3 \cdot 3 \cdot 3 = 27$
4	$3^4 = 81$
5	$3^5 = 81 \cdot 3 = 243$
6	$3^6 = 3^5 \cdot 3 = 243 \cdot 3 = 729$

$*3$ $*3$ $*3$ $*3$

for non-zero base $(b \neq 0)$:

zero exponent: $b^0 = 1$

negative exponents: $b^{-n} = \frac{1}{b^n}$

Ex: $3^{-5} = \frac{1}{3^5} = \frac{1}{243}$

$\div 3$

$*3$

"exponential growth"

WW: "Integer Exponents"

$$b^{-1} = \frac{1}{b^1} = \frac{1}{b}$$

#1) a) $17^{-1} = \frac{1}{17^1} = \boxed{\frac{1}{17}}$

b) $\left(\frac{1}{9}\right)^{-1} = \frac{1}{(1/9)^1} = \frac{1}{(1/9)} = 1\cdot\left(\frac{9}{1}\right) = \boxed{9}$

(shortcut: $\left(\frac{m}{n}\right)^{-1} = \frac{1}{(m/n)^1} = \frac{1}{(m/n)} = 1\cdot\frac{n}{m} = \frac{n}{m}$

any fraction raised to "-1" exponent just flips the fraction!

c) $\left(\frac{11}{7}\right)^{-1} = \boxed{\frac{7}{11}}$

#2) (b) $-(12^{-2}) = -\left(\frac{1}{12^2}\right) = -\frac{1}{144}$

different! $(-12)^{-2} = \frac{1}{(-12)^2} = \frac{1}{144}$ (similarly: $(-6)^{-2} = \frac{1}{(-6)^2} = \frac{1}{36}$

Simplifying exponents

In general:

$b^m \cdot b^n = b^{m+n}$

$\frac{b^m}{b^n} = b^{m-n}$

$(b^n)^m = b^{n \cdot m}$

#3) (a) $x^5 \cdot x^6 = x^{11}$

$\underbrace{(x \cdot x \cdot x \cdot x \cdot x)}_{5 \text{ factors}} \underbrace{(x \cdot x \cdot x \cdot x \cdot x \cdot x)}_{6 \text{ factors}}$

OpenStax Examples 5.12, 5.13

(b) $n^5 \cdot n = n^5 \cdot n^1 = n^{5+1} = n^6$

(c) $\frac{x^8}{x^{10}} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \boxed{\frac{1}{x^2} = x^{-2}}$

(d) $\frac{n}{n^6} = \frac{n^1}{n^6} = n^{1-6} = n^{-5} = \boxed{\frac{1}{n^5}}$

#4) $(a^9)^3 = (a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a)^3 = \underbrace{(a \cdots a)}_{9 \text{ facto}} \underbrace{(a \cdots a)}_{9} \underbrace{(a \cdots a)}_{9}$

$= a^{27}$

"nested exponents"

#6)

(d) $\cancel{q^{-8} q^{-5}}$ $q^{-5} \cdot q^{-8} = \frac{1}{q^5} \cdot \frac{1}{q^8} = \frac{1}{q^{13}}$

$= q^{-5-8} = q^{-13} = \frac{1}{q^{13}}$

"Fractional Exponents"

→ WebWork: "Exponential Function"

#3) $216^{1/3} = \sqrt[3]{216}$

$= 6$

(b/c $6^3 = 216$)

$b^{1/3} = \sqrt[3]{b}$

"cube root of b"

$\frac{1}{2}$ power represents square root!

$36^{1/2} = \sqrt[2]{36} = 6$

$b^{1/2} = \sqrt{b} = \sqrt[2]{b}$

(why? so that:

$b^{1/2} \cdot b^{1/2} = b^{1/2+1/2} = b^1 = \underline{b}$

$\sqrt{b} \cdot \sqrt{b} = b$

Class #38 - Thurs, Dec 2

<u>Last time</u>: exponents (mostly review?)

- positive integer exponents $7^3 = 7 \cdot 7 \cdot 7$
- negative integer exponents: $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$
- "zero exponents":

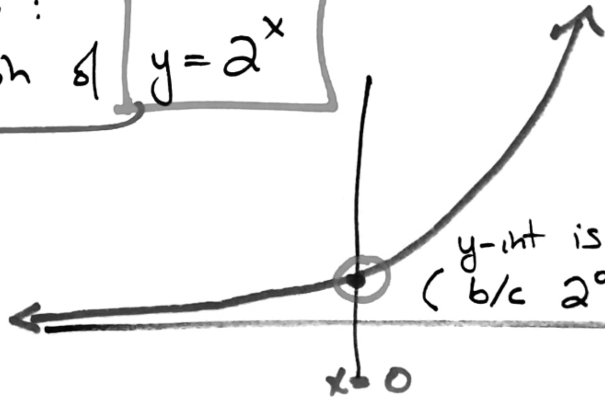
$$b^0 = 1$$ (for any non-zero base b, ie. $b \neq 0$)

$$\boxed{2^0 = 1}$$

Therefore: graph of $\boxed{y = 2^x}$

y-int? plug in $x = 0$:

$$y = 2^0 = 1$$



when $x=0$, $y = 2^0 = 1$

y-int is at $\boxed{(0,1)}$ (b/c $2^0 = 1$)

"Fractional (or rational) exponents"

$68^{1/2} = \sqrt{68} = \sqrt{4(17)} = \boxed{2\sqrt{17}}$
$\approx 8.24\ldots$

$125^{1/3} = \sqrt[3]{125} = 5$
(b/c $5^3 = 125$)

Def'n:
$b^{1/n} = \underbrace{\sqrt[n]{b}}_{\text{"}n^{th}\text{ root of } b\text{"}}$

close to $\sqrt{81} = 9$

$80^{1/2} = \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$
$\approx 8.94\ldots$

Simple "exponential equations":

Ex: Solve $2^{x} = 512$

$2^{9} = 512 \implies \boxed{x = 9}$

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
16
32
64
128
256
512

Class # 39 - Fri Dec 3

Last time (yesterday)

- reviewed exponents $\longrightarrow$ "exponential functions"

$\underbrace{\text{reviewed exponents}}$

positive integer exponents $\quad b^n = \underbrace{b \cdot b \cdot b \cdots b}_{n \text{ factors}}$

$\Rightarrow$ negative " "

$\Rightarrow$ fractional (rational) exponents $\quad b^{-n} = \dfrac{1}{b^n}$

$\underbrace{32^{1/5}}_{\text{solve the eqn}} = \sqrt[5]{32} = ?$ $\Longleftarrow$ $b^{1/n} = \sqrt[n]{b}$

solve the eqn

$x^5 = 32$

$\Rightarrow \underline{\underline{x=2}}$

$b^{m/n}$

Ex: $\underbrace{2^{5/2}}_{b/c\ \frac{5}{2} = \frac{1}{2} \cdot 5} = (2^{1/2})^5 = (2^5)^{1/2}$

Example: Exponents (powers of 2) and computer memory

x	2^x
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
	$2^3 = 8$
	$2^4 = 16$
	$2^5 = 32$
	$2^6 = 64$
	$2^7 = 128$
	$2^8 = 256$
	$2^9 = 512$
	$2^{10} = 1024$

$2^{10} = 1,024$

1 byte $= 8$ bits $= 2^3$ bits

1 kb $= 1024$ bytes $= 2^{10}$ $\underbrace{\text{bytes}}_{2^3 \text{ bits}}$ $= 2^{10} \cdot 2^3$ bits

1 mb $= 1024$ kb $= 2^{10}(2^{10} \cdot 2^3)$ bits $= 2^{23}$ bits

1 gb $= 1024$ mb $= 2^{10}$ mb $= 2^{10}(2^{23}$ bits$) = 2^{33}$ bits

1 tb $= 1024$ gb $= 2^{10}$ gb $= 2^{10}(2^{33}$ bits$)$

$= 2^{43}$ bits

$= 2^{40}$ bytes

b/c 1 byte $= 2^3$ bits

$\approx 1.0995 \times 10^{12}$

≈ 1.1 trillion bytes

$\boxed{\text{1 TB HD is} \sim \$50!}$

$10^{12} = 1,000,000,000,000 = 1$ trillion

Last time : simple exponential equations

(a) $8^{x} = 2$ (since $8 = 2^3$, $2 = \sqrt[3]{8} = 8^{1/3}$

what is the x-value where $8^x = 2$?

$\underline{x = 1/3}$

hint : x is a fraction!

(b) $8^{x} = \frac{1}{64} = \frac{1}{8^2} = 8^{-2}$

x-value such that $y = 8^x = \frac{1}{64}$?

hint : y is a negative integer.

$x = ?$

$\boxed{x = -2}$

Now we will look at (define) "logarithmic" functions..

(why? b/c they will allow us to solve equations such as

$8^x = 10$ } we could use a calculator...

Logarithmic functions

$y = \log_b x$ (log function with base b)

$\underbrace{}$

means $b^y = x$

Rewrite in "exponential form"

$\log_{10}(0.01) = -2$

$\Rightarrow 10^{-2} = 0.01 = \frac{1}{100} = \frac{1}{10^2}$

Ex: $\log_2 x$

$\log_2 16 = \boxed{?} \iff 2^? = 16$

$2^4 = 16$

$\Rightarrow \log_2 16 = 4$

$\log_3(243) = \boxed{5}$

$3^{\boxed{5}} = 243$