Class #19 – Thurs, Oct. 14

Last time?

- Examples from "Quadratic Formula" WW

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ — "discriminant"

2 solutions of $ax^2 + bx + c = 0$

(Based on the sign of the discriminant $b^2 - 4ac$:)

(1) $b^2 - 4ac > 0$, then the QF gives 2 real #s

(2) $b^2 - 4ac < 0$, then the QF gives 2 "complex" #s

(3) $b^2 - 4ac = 0$, then the QF gives 1 real #

Examples

(1) Solutions of "$8x^2 + 15x + 6 = 0$" are the 2 real (irrational) #s :

$$x = \frac{-15 \pm \sqrt{33}}{16}$$

(2) Solutions of "$x^2 - 14x + 98 = 0$" are the 2 "complex" #s :

$$x = \frac{14 \pm \sqrt{-196}}{2} = 7 \pm 7i$$

(3) Solutions of "$x^2 + 6x + 9 = 0$" is

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(9)}}{2(1)} = \frac{-6 \pm \sqrt{36-36}}{2} = \frac{-6 \pm \sqrt{0}}{2}$$

$$= \frac{-6 \pm 0}{2} = \frac{-6}{2} = -3$$

Solve by factoring :

$$(x+3)(x+3) = 0 \implies x+3 = 0$$

What are "complex numbers"? (§8.8 of OpenStax)

- allow us to deal with square roots of negative #s

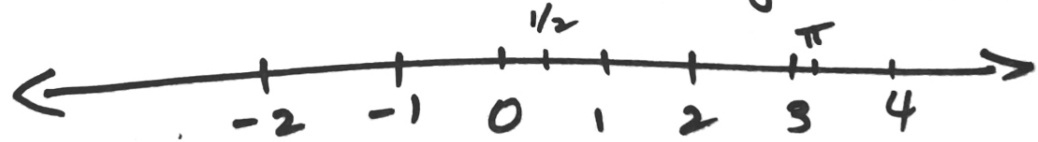
E.g. $\sqrt{-196} = ?$

- we use a new number symbol "i" ← "basic imaginary" # to represent the square root of -1:

$$i = \sqrt{-1}$$

(why do we need this new "imaginary" # i for $\sqrt{-1}$? b/c there are no real # square roots of negative #s!

$\mathbb{R}$



(what is $\sqrt{-1}$? it's a # such that when you square it, you get -1

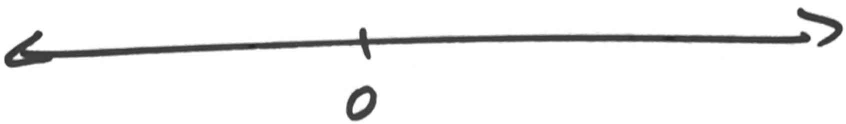
What is $\sqrt{-1}$?

By definition of $\sqrt{\ }$,

no such real #

$\sqrt{-1}$ is some # such that when you square it, you get -1 ...

note that $(-1)(-1) = 1$, so $\sqrt{-1} \neq -1$



there's no way to get a negative #
from squaring a real # !

So we introduce a new # "i" which is
defined by: $i = \sqrt{-1}$

Imaginary #s

We have defined:

$$i = \sqrt{-1} \quad (\text{ie, } i^2 = -1)$$

which allows us to represent the square root of any negative #:

Ex: $\sqrt{-196} = \sqrt{(-1)\cdot(196)}$

$= \sqrt{-1}\cdot\sqrt{196}$

$= (i)\cdot(14)$

$= 14i$

property of square roots: $\sqrt{a\cdot b} = \sqrt{a}\cdot\sqrt{b}$

Check? $(14i)^2 = (14i)\cdot(14i) = (14)(14)(i)(i)$
$= 196i^2 = -196$

Complex #s

Ex: $7 + 7i$

$7 - 7i$

we got these as solutions to "$x^2 - 14x + 98 = 0$":

$$x = \frac{14 \pm \sqrt{-196}}{2} = \frac{14 \pm 14i}{2} = \boxed{7 \pm 7i}$$

Def'n: A complex number is a number of the form

"$a + bi$"

where a and b are any real #s.