Class ~~#~~ #16 - Thurs, Oct 7

- **Exam #1** : due tomorrow (Fri) - 5pm.
  - discussed the exam at end of class on Tues.

  → "Class Notes - Exam #1"

Today :

- Solving quadratic equations by "completing the square" (and then applying the square root property)

⟹ WW: "SquareRootProperty" #5-9

Thus far : we have been able to solve certain types of quadratic equation using the SRP :

Solve : $(x-h)^2 = k.$

$\stackrel{\sqrt{\ }}{\Longrightarrow}$ $x - h = \pm\sqrt{k}$ $+h$

$+h$

$\Longrightarrow$ $\boxed{x = h \pm \sqrt{k}}$

Ex : $(x-4)^2 = 17$

in std form :

$(x-4)(x-4) = 17$

$x^2 - 8x + 16 = 17$

$x^2 - 8x - 1 = 0$

Idea 1 "completing the square" :

starting w/ $x^2 - 8x - 1 = 0$,

do the necessary algebra to put it in the form $(x-4)^2 = 17$

(so that we can apply the SRP !)

Example ("completing the square")

Say we want to solve the quadratic eqn

$$x^2 + 8x + 1 = 0$$

(in std form: $ax^2 + bx + c = 0$).

$\underline{x^2 + bx + c = 0}$

1. Move the constant term c to RHS:

$$x^2 + 8x \underbrace{+16} = -1 \; +16$$

$\left(\frac{8}{2}\right)^2 = 4^2 = 16$

add a constant # so that the LHS is a perfect square quadratic

add the same # to RHS (to keep the equation in balance)

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

Add $\left(\frac{b}{2}\right)^2$ so that LHS is a perfect square quadratic.

2. Factor the LHS as "perfect square"

$$\left(\frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)\left(\frac{b}{2}\right) = \frac{b^2}{4}$$

Recap:

Specific example:

$$x^2 + 8x + 1 = 0$$

"complete the square"

$$x^2 + 8x + 16 = -1 + 16$$

$$\left(\frac{8}{2}\right)^2 = 4^2 = 16$$

Factor LHS (a perfect square):

$$(x+4)^2 = 15$$

Solve for x (using SRP):

$$\sqrt{(x+4)^2} = \pm\sqrt{15}$$

$$x + 4 = \pm\sqrt{15}$$

$$\boxed{x = -4 \pm \sqrt{15}}$$

In general:

$$1x^2 + bx + c = 0.$$

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \underbrace{-c + \left(\frac{b}{2}\right)^2}_{\frac{b^2}{4} - c}$$

$$\left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = \frac{b^2}{4} - \frac{c}{1} \cdot \frac{4}{4}$$

$$\sqrt{\left(x + \frac{b}{2}\right)^2} = \sqrt{\frac{b^2 - 4c}{4}}$$

$$x + \frac{b}{2} = \pm\sqrt{\frac{b^2 - 4c}{4}}$$

$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{\sqrt{4}}$$

So : by "completing the square" and using the Square Root Property, we have found that solutions of the quadratic equation

$$x^2 + bx + c = 0$$

are

$$x = -\frac{b}{2} \pm \frac{\sqrt{b^2-4c}}{2}$$

i.e.,

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4c}}{2}}$$

quadratic formula for $x^2 + bx + c = 0$ (i.e., $a = 1$)

For $ax^2 + bx + c = 0$,

full quadratic formula :

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.}$$

WW, "SRP" #6

$$w^2 - 3w + \frac{9}{4}$$

$$= \left(w - \frac{3}{2}\right)^2$$

$b = -3$

$\frac{b}{2} = -\frac{3}{2}$

$\left(\frac{b}{2}\right)^2 = \left(-\frac{3}{2}\right)^2$

$= \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)$

$= \frac{3 \cdot 3}{2 \cdot 2} = \frac{9}{4}$

Check (FOIL):

$$\left(w - \frac{3}{2}\right)^2 = \left(w - \frac{3}{2}\right)\left(w - \frac{3}{2}\right)$$

$$= w^2 - \frac{3}{2}w - \frac{3}{2}w + \frac{9}{4}$$

$$= w^2 - 3w + \frac{9}{4} \checkmark$$