Course: MAT1275 Date: Mon, 03/09/20
Instructor: N. Ovshey

Sample Exam #1 Solutions

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1) Add/Subtract the rational expressions and simplify your answer (8 pts.):

Steps to adding/subtracting rational expressions:
1) factor each denom.
2) Use factorizations to find LCD
3) convert each fraction to equivalent one with denom = LCD
4) add numerators, factor final numerator and cancel any common factors with denom., if possible

a) $\frac{5}{9-x^2}-\frac{4}{x^2+4x+3}=$

First denom: $9-x^2=-(x^2-9)=-(x-3)(x+3)$
Second denom: $x^2+4x+3=(x+3)(x+1)$
LCD (make positive): $(x+3)(x-3)(x+1)$

First fraction: $\frac{5}{9-x^2}=\frac{5}{-(x-3)(x+3)}=\frac{5\cdot(-1)\cdot(x+1)}{-(x-3)(x+3)\cdot(-1)\cdot(x+1)}=\frac{-5(x+1)}{(x-3)(x+3)(x+1)}=\frac{-5x-5}{(x-3)(x+3)(x+1)}$

Second Fraction: $-\frac{4}{x^2+4x+3}=\frac{-4\cdot(x-3)}{(x+3)(x+1)\cdot(x-3)}=\frac{-4(x-3)}{(x+3)(x+1)(x-3)}=\frac{-4x+12}{(x+3)(x+1)(x-3)}$

Solution: $\frac{5}{9-x^2}-\frac{4}{x^2+4x+3}=\frac{5}{9-x^2}+\frac{-4}{x^2+4x+3}=\frac{-5x-5}{(x-3)(x+3)(x+1)}+\frac{-4x+12}{(x+3)(x+1)(x-3)}=\boxed{\frac{\mathbf{-9x+7}}{\mathbf{(x+3)(x+1)(x-3)}}}$

Note that in this case the last faction cannot be simplified any further (numerator is prime)

b) $\frac{2}{3x-15}+\frac{x}{25-x^2}=\frac{2}{3(x-5)}+\frac{x}{-(x^2-25)}=\frac{2}{3(x-5)}+\frac{-x}{(x-5)(x+5)}=\frac{2\cdot(x+5)}{3(x-5)\cdot(x+5)}+\frac{-x\cdot 3}{(x-5)(x+5)\cdot 3}=$

$$=\frac{2\cdot(x+5)+-3x}{3(x-5)\cdot(x+5)}=\frac{2x+10-3x}{3(x-5)\cdot(x+5)}=\boxed{\frac{\mathbf{-x+10}}{\mathbf{3(x-5)(x+5)}}}$$

Again, the final fraction cannot be simplified further.

2) Simplify the complex fraction (8 pts.):

Steps to simplifying complex fractions:
1) identify all denoms.
2) find LCD
3) multiply num. and denom. of complex fraction by LCD

a) $\frac{\frac{2}{y^2}+\frac{1}{y}}{\frac{4}{y^2}-\frac{1}{y}}=$

LCD of y, y^2 is y^2

$$\frac{\frac{2}{y^2}+\frac{1}{y}}{\frac{4}{y^2}-\frac{1}{y}} = \frac{\left(\frac{2}{y^2}+\frac{1}{y}\right)\cdot y^2}{\left(\frac{4}{y^2}-\frac{1}{y}\right)\cdot y^2} = \boxed{\mathbf{\frac{2+y}{4-y}}}$$

b) $\frac{\frac{2}{x}+\frac{1}{y}}{\frac{3}{y}-\frac{4}{x}} = \frac{\left(\frac{2}{x}+\frac{1}{y}\right)\cdot xy}{\left(\frac{3}{y}-\frac{4}{x}\right)\cdot xy} = \boxed{\mathbf{\frac{2y+x}{3x-4y}}}$

Solve rational equation and check your answer (13 pts. each):

<u>Steps to solving rational equations</u>:
1) Factor all denoms
2) find LCD
3) multiply both sides by LCD; don't forget to distribute
4) solve resulting linear or quadratic equation
5) check each solution obtained in step 4 in original equation and keep only those that check out

3) $\frac{x}{x+6} = \frac{72}{x^2-36} + 4$

LCD of $x+6, x^2-36$: $x^2-36 = (x+6)(x-6)$

Multiply both sides by LCD: $(x-6)(x+6)$

$$(x-6)(x+6)\cdot\left(\frac{x}{x+6}\right) = (x-6)(x+6)\cdot\left(\frac{72}{x^2-36}+4\right)$$

$$(x-6)x = 72 + 4(x-6)(x+6)$$

$$x^2-6x = 72+4x^2-144 = 4x^2-72$$

$$3x^2+6x-72 = 0$$

$$3(x^2+2x-24) = 3(x+6)(x-4) = 0$$

$$x = -6, 4$$

Check each solution in <u>original equation</u>. You will that only 4 checks out, so the $\boxed{\textbf{actual solution set is } \{4\}}$.

4) $\frac{h}{2} - \frac{h}{h-4} = \frac{-4}{h-4}$

Note: the original question had a typo in it. The left side should be negative as corrected above.

LCD of 2, $(h-4)$ is $2(h-4)$

Multiply both sides by LCD:

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4) (cont'd)

$$2(h-4)\cdot\left(\frac{h}{2} - \frac{h}{h-4} = \frac{-4}{h-4}\right)$$

$$2(h-4)\cdot\left(\frac{h}{2}\right) - 2(h-4)\cdot\left(\frac{h}{h-4}\right) = 2(h-4)\cdot\left(\frac{-4}{h-4}\right)$$

$$(h-4)h - 2h = -8$$

$$h^2 - 4h - 2h = -8$$

$$h^2 - 6h + 8 = 0$$

$$(h-4)(h-2) = 0$$

So the potential solution set is $\{4,2\}$. Checking each one shows that 4 IS NOT a solution b/c it makes denom. Zero, and $h = 2$ DOES satisfy the original rational equation. Thus the $\boxed{\textbf{actual solution set is } \{\mathbf{2}\}}$.

Here's the check for $h = 2$. Make sure you do this work on the exam.

$$\frac{2}{2} - \frac{2}{2-4} \stackrel{?}{=} \frac{-4}{2-4}$$

$$1 - \frac{2}{-2} \stackrel{?}{=} \frac{-4}{-2}$$

$$1 - (-1) \stackrel{?}{=} 2$$

$$2 \stackrel{true}{=} 2$$

5) (8 pts.) a) Write $x^{4/7}$ in radical notation:

$$x^{\frac{4}{7}} \stackrel{\text{def}}{=} \boxed{\sqrt[7]{x^4}}$$

b) Simplify expression using properties of fractional exponents. Write final answer using positive exponents only: $\left(\frac{50p^{-1}q}{2pq^{-3}}\right)^{1/2} =$

Simplify inside (base) first: $\frac{50p^{-1}q}{2pq^{-3}} = \frac{25q\cdot q^3}{p\cdot p^1} = \frac{25q^4}{p^2}$

Raise to $\frac{1}{2}$ power: $\left(\frac{25q^4}{p^2}\right)^{\frac{1}{2}} = \frac{(25)^{\frac{1}{2}}\cdot(q^4)^{\frac{1}{2}}}{(p^2)^{\frac{1}{2}}} = \frac{5\cdot q^{4\cdot\frac{1}{2}}}{p^{2\cdot\frac{1}{2}}} = \frac{5q^2}{p^1} = \boxed{\frac{\mathbf{5q^2}}{\mathbf{p}}}$

6) Simplify radicals (8 pts.): a) $\sqrt{64m^5n^{20}}$ = b) $\sqrt[4]{32p^8qr^5}$ =

Factor radicands to primes and write each prime factor as the highest power divisible by the index times whatever is left over to make the radicand:

a) $64m^5n^{20} = 2^6m^4 \cdot m^1n^{20}$

$$\sqrt{64m^5n^{20}} = \sqrt{2^6m^4 \cdot m^1n^{20}} = 2^3 \cdot m^2 \cdot n^{10}\sqrt{m^1} = \boxed{\mathbf{8m^2n^{10}\sqrt{m}}}$$

b) $32p^8qr^5 = 2^5p^8qr^5 = 2^4 \cdot 2 \cdot p^8 \cdot q \cdot r^4 \cdot r$

$$\sqrt[4]{32p^8qr^5} = \sqrt[4]{2^4 \cdot 2 \cdot p^8 \cdot q \cdot r^4 \cdot r} = 2 \cdot p^2 \cdot r\sqrt[4]{2 \cdot q \cdot r} = \boxed{\mathbf{(2p^2r)\sqrt[4]{2qr}}}$$

Add/Subtract. Simplify radical expressions as much as possible

7) (8 pts.) a) $2\sqrt{12} + \sqrt{48}$ = b) $2s^2\left(\sqrt[3]{s^2t^6}\right) + 3t^3\left(\sqrt[3]{8s^8}\right)$ =

Simplify each radical first.

a) $2\sqrt{12} + \sqrt{48} = 2\sqrt{4\cdot 3} + \sqrt{16 \cdot 3} = 2 \cdot 2\sqrt{3} + 4\sqrt{3} = 4\sqrt{3} + 4\sqrt{3} = \boxed{\mathbf{8\sqrt{3}}}$

b) $2s^2\left(\sqrt[3]{s^2t^6}\right) + 3t^3\left(\sqrt[3]{8s^8}\right) = 2s^2 \cdot t^2\left(\sqrt[3]{s^2}\right) + 3t^3 \cdot 2 \cdot s^2\left(\sqrt[3]{s^2}\right) =$
$= 2s^2t^2\left(\sqrt[3]{s^2}\right) + 6t^3s^2\left(\sqrt[3]{s^2}\right) = \boxed{\mathbf{(2s^2t^2 + 6t^3s^2)\left(\sqrt[3]{s^2}\right)}}$

8) (8 pts.) Multiply and simplify: a) $(3 + \sqrt{2})(\sqrt{2} + 5)$ =

FOIL out the product:

$$(3 + \sqrt{2}) \cdot (\sqrt{2} + 5) = 3 \cdot \sqrt{2} + 3 \cdot 5 + (\sqrt{2}) \cdot (\sqrt{2}) + (\sqrt{2}) \cdot 5 = 3\sqrt{2} + 15 + 2 + 5\sqrt{2} = \boxed{\mathbf{17 + 8\sqrt{2}}}$$

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8) (cont'd)

b) Rationalize denominator of quotient: $\frac{3+\sqrt{2}}{\sqrt{2}-5} =$

First, determine the radical conjugate of the denominator, then multiply num. and denom. of given fraction by it and simplify.

Radical conjugate of $\sqrt{2}-5$ is $\sqrt{2}+5$.

$$\frac{3+\sqrt{2}}{\sqrt{2}-5} = \frac{(3+\sqrt{2})\cdot(\sqrt{2}+5)}{(\sqrt{2}-5)\cdot(\sqrt{2}+5)} = \frac{17+8\sqrt{2}}{(\sqrt{2})^2-(5)^2} = \frac{17+8\sqrt{2}}{2-25} = \boxed{-\frac{\mathbf{17+8\sqrt{2}}}{\mathbf{23}}}$$

Solve and check the following radical equations (13 pts each):

<u>Steps to solving a radical equation</u>:

1) isolate radical (if there are two then separate them, i.e., move one to another side; it doesn't matter which)
2) raise both sides to the index of the radical(s)
3) solve resulting linear, quadratic or another radical equation (for this last case, repeat steps 1 & 2)
4) check the potential solutions obtained in step 3 in original radical equation. The solutions that check out form the actual solution set.

9) x – $\sqrt{10-3x}$ = 2

<u>Step 1 (isolate radical)</u>: $x - \sqrt{10-3x} = 2 \xRightarrow{subtract\ x\ from\ both\ sides} -\sqrt{10-3x} = 2-x$

<u>Step 2 (raise both sides to 2^{nd} power)</u>: $-\sqrt{10-3x} = 2-x \xRightarrow{square\ both\ sides} \left(-\sqrt{10-3x}\right)^2 = (2-x)^2$

$$(-1)^2\cdot\left(\sqrt{10-3x}\right)^2 = (2-x)\cdot(2-x)$$

$$1\cdot(10-3x) = (2)^2 - 2\cdot(2)\cdot x + x^2$$

$$10-3x = 4-4x+x^2$$

9) (cont'd)

Step 3 (solve equation obtained in step 2):

$$10-3x=4-4x+x^2 \xRightarrow{\text{collect all terms on left side}} x^2-4x+4-10+3x=0$$

$$x^2-x-6=0 \xRightarrow{\text{factor left side}} (x-3)(x+2)=0 \xRightarrow{\text{solve for } x} \text{potential solution set is } \{3,-2\}$$

Step 4 (check each potential solution):

Check $x=3$:

$$3-\sqrt{10-3\cdot 3} \overset{?}{\cong} 2$$
$$3-\sqrt{10-9} \overset{?}{\cong} 2$$
$$3-\sqrt{1} \overset{?}{\cong} 2$$
$$3-1 \overset{true}{\cong} 2$$

Check $x=-2$

$$(-2)-\sqrt{10-3\cdot(-2)} \overset{?}{\cong} 2$$
$$-2-\sqrt{10+6} \overset{?}{\cong} 2$$
$$-2-\sqrt{16} \overset{?}{\cong} 2$$
$$-2-4 \overset{false}{\cong} 2$$

Thus $\boxed{\textbf{actual solution set is } \{3\}}$

10) x = $\sqrt{4x}-1$

$$x=\sqrt{4x}-1 \xRightarrow{\text{isolate radical}} (x+1)=\sqrt{4x} \xRightarrow{\text{square both sides}} (x+1)^2=\left(\sqrt{4x}\right)^2$$

$$x^2+2\cdot x\cdot 1+1^2=4x$$

$$x^2+2x+1=4x \xRightarrow{\text{collect all terms on left side}} x^2+2x+1-4x=0$$

$$x^2-2x+1=0 \xRightarrow{\text{factor left side}} (x-1)^2=0$$

Potential solution set is {1}. Now check $x=1$ in original radical equation.

Check $x=1$

$$1 \overset{?}{\cong} \sqrt{4\cdot 1}-1$$
$$1 \overset{true}{\cong} 2-1$$

So $\boxed{\textbf{actual solution set is } \{1\}}$

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11) Solve the following 3 x 3 system of equations. Write your solution as an ordered triple.

$$2x - 3y + z = -9$$
$$3x + 5y + 2z = 16$$
$$-4x + 2y - 3z = 4$$

<u>Solution</u>:

First, number the three given equations:

$$1)\ 2x - 3y + z = -9$$
$$2)\ 3x + 5y + 2z = 16$$
$$3)\ -4x + 2y - 3z = 4$$

Second, find the variable with a coefficient of 1 or -1 and eliminate that variable by using two different pairs of original equations. I will eliminate the z variable.

$1)\ 2x - 3y + z = -9 \xrightarrow{mult.\ by\ -2} -4x + 6y - 2z = 18$

$2)\ 3x + 5y + 2z = 16 \xrightarrow{mult.\ by\ 1} 3x + 5y + 2z = 16$

Add the resulting equations and call the sum equation A).

$$A)\ -x + 11y = 34$$

Now, combine equations 1) & 2):

$1)\ 2x - 3y + z = -9 \xrightarrow{mult.\ by\ 3} 6x - 9y + 3z = -27$

$3)\ -4x + 2y - 3z = 4 \xrightarrow{mult.\ by\ 1} -4x + 2y - 3z = 4$

Add and call the sum equation B).

$$B)\ 2x - 7y = -23$$

Solve the A, B system for x,y:

$A)\ -x + 11y = 34 \xrightarrow{mult.\ by\ 2} -2x + 22y = 68$

$B)\ 2x - 7y = -23 \xrightarrow{mult.\ by\ 1} 2x - 7y = -23$

Add to get $15y = 45$. So $y = \frac{14}{15} = 3$. Substitute this value for y in A) and solve for x: $-x + 11(3) = 34 \rightarrow -x + 33 = 34 \rightarrow -x = 1 \rightarrow x = -1$.

Finally, substitute $x = -1, y = 3$ into original equation 1) and solve for z:

$$2(-1) - 3(3) + z = -9 \rightarrow -2 - 9 + z = -9 \rightarrow -11 + z = -9 \rightarrow z = -9 + 11 = 2.$$

$\boxed{The\ solution\ is\ there\ ordered\ triple: (-1, 3, 2)}$.