Lesson #11

MAT 1372 Statistics with Probability

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Mean

Mean of a discrete probability distribution

- $\mu = \Sigma x P(x)$
- Each value of x is multiplied by its corresponding probability and the products are added.

Example: Finding the Mean

The probability distribution for the personality inventory test for passive-aggressive traits is given. Find the mean score.

Solution:

x	$P(x)$
1	0.16
2	0.22
3	0.28
4	0.20
5	0.14

$$\mu = \Sigma xP(x) = 2.94$$

Variance and Standard Deviation

Variance of a discrete probability distribution

- $\sigma^2 = \Sigma(x - \mu)^2 P(x)$

Standard deviation of a discrete probability distribution

- $\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x-\mu)^2 P(x)}$

Example: Finding the Variance and Standard Deviation

The probability distribution for the personality inventory test for passive-aggressive traits is given. Find the variance and standard deviation. ($\mu = 2.94$)

x	$P(x)$
1	0.16
2	0.22
3	0.28
4	0.20
5	0.14

Solution: Finding the Variance and Standard Deviation

Recall $\mu = 2.94$

x	$P(x)$	$x - \mu$
1	0.16	$1 - 2.94 = -1.94$
2	0.22	$2 - 2.94 = -0.94$
3	0.28	$3 - 2.94 = 0.06$
4	0.20	$4 - 2.94 = 1.06$
5	0.14	$5 - 2.94 = 2.06$

Variance: $\sigma^2 = \Sigma(x - \mu)^2 P(x) = 1.616$

Standard Deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{1.616} \approx 1.3$

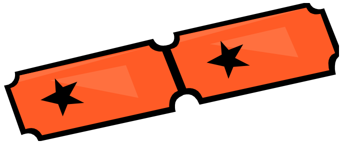
Expected Value

Expected value of a discrete random variable

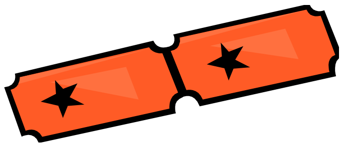
- Equal to the mean of the random variable.
- $E(x) = \mu = \Sigma x P(x)$

Example: Finding an Expected Value

At a raffle, 1500 tickets are sold at $2 each for four prizes of $500, $250, $150, and $75. You buy one ticket. What is the expected value of your gain?



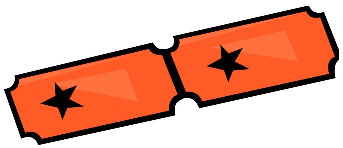
Solution: Finding an Expected Value



- To find the gain for each prize, subtract the price of the ticket from the prize:
  - Your gain for the \$500 prize is \$500 – \$2 = \$498
  - Your gain for the \$250 prize is \$250 – \$2 = \$248
  - Your gain for the \$150 prize is \$150 – \$2 = \$148
  - Your gain for the \$75 prize is \$75 – \$2 = \$73
- If you do not win a prize, your gain is \$0 – \$2 = –\$2

Solution: Finding an Expected Value

- Probability distribution for the possible gains (outcomes)



Gain, x	\$498	\$248	\$148	\$73	–\$2
P(x)	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1496}{1500}$

$$
\begin{aligned}
E(x) &= \Sigma x P(x) \\
&= \$498 \cdot \frac{1}{1500} + \$248 \cdot \frac{1}{1500} + \$148 \cdot \frac{1}{1500} + \$73 \cdot \frac{1}{1500} + (-\$2) \cdot \frac{1496}{1500} \\
&= -\$1.35
\end{aligned}
$$

You can expect to lose an average of \$1.35 for each ticket you buy.