form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_3x^3 + a_2x^2 + a_1x + a_0,$$

where n is a nonnegative integer, x is a variable,* and each of $a_0, a_1, \ldots, a_n$ is a constant, called a **coefficient.** The coefficient a_0 is called the **constant term.** A polynomial that consists only of a constant term, such as 12, is called a **constant polynomial.** The **zero polynomial** is the constant polynomial 0.

The *exponent* of the highest power of x that appears with *nonzero* coefficient is the **degree** of the polynomial, and the nonzero coefficient of this highest power of x is the **leading coefficient.** For example,

Polynomial	**Degree**	**Leading Coefficient**	**Constant Term**
$6x^7 + 4x^3 + 5x^2 - 7x + 10$	7	6	10
x^3	3	1	0
12 (think of this as $12x^0$)	0	12	12
$0x^9 + 2x^6 + 3x^7 + x^8 - 2x - 4$	8	1	−4

The degree of the zero polynomial is *not defined* since no exponent of x occurs with nonzero coefficient.

The Division Algorithm

If a polynomial $f(x)$ is divided by a nonzero polynomial $h(x)$, then there is a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$ such that

Dividend = Divisor · Quotient + Remainder

$$f(x) = h(x)\,q(x) + r(x),$$

where either $r(x) = 0$ or $r(x)$ has degree less than the degree of the divisor $h(x)$.

Remainders and Factors

The remainder in polynomial division is 0 exactly when the divisor is a factor of the dividend. In this case, the quotient is the other factor.

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - c$, then the remainder is the number $f(c)$.

Factor Theorem

The number c is a root of the polynomial $f(x)$ exactly when $x - c$ is a factor of $f(x)$.

Handout for Section
4.2 Polynomial Functions

Number of Roots

A polynomial of degree n has at most n distinct roots.