

$$\textcircled{1} \quad f(x) = x^2 - 10x + 10$$

$$f'(x) = 2x - 10 = 0 \quad \frac{2x = 10}{2} \quad x = 5$$

$$\text{critical point } c = \boxed{5}$$

$$f(5) = 25 - 50 + 10 = -15$$

$$f(c) = \boxed{-15}$$

on $[0, 10]$

$$f(0) = \boxed{10}$$

$$f(10) = 100 - 100 + 10 = \boxed{10}$$

$$\text{Minimum value} = \boxed{-15}$$

$$\text{Maximum value} = \boxed{10}$$

on $[0, 1]$ we do not have any critical point

$$f(0) = 10 \quad f(1) = 1 - 10 + 10 = 1$$

$$\text{Minimum value} = \boxed{1}$$

$$\text{Maximum value} = \boxed{10}$$

$$\textcircled{2} \quad f(x) = 4x^2 + 3x - 12$$

$$\text{Average slope in } [-1, 1] \text{ is } \frac{f(1) - f(-1)}{1 - (-1)}$$

$$f(1) = 4 + 3 - 12 = -5$$

$$f(-1) = 4(-1)^2 + 3(-1) - 12 = -11$$

$$\frac{f(1) - f(-1)}{2} = \frac{-5 - (-11)}{2} = \frac{-5 + 11}{2} = \frac{6}{2} = 3 \quad \bar{m} = \boxed{3}$$

$$f'(x) = 8x + 3$$

$$\frac{8x + 3}{3} = \frac{3}{3} \quad \frac{8x}{8} = \frac{0}{8} \quad x = 0 \quad c = \boxed{0}$$

$$\textcircled{3} \quad f(150) = 31 \quad f'(150) = 3 \quad \text{Estimate } f(155)$$

$f(155)$ is approximately

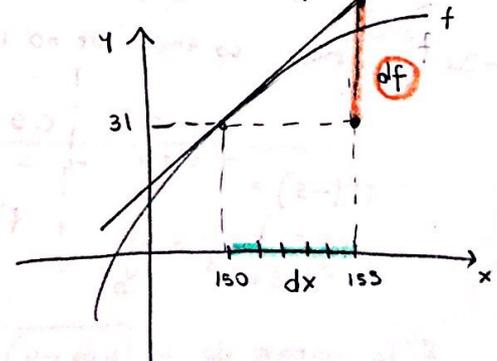
$$f(150) + df$$

$$df = f'(150) dx \quad dx = 155 - 150 = 5$$

$$f(155) \approx f(150) + f'(150)(155 - 150)$$

$$= 31 + 3(5) = 31 + 15 = \boxed{46}$$

Idea from differentials



$$y = \sqrt{6-x}$$

$$dy = f'(x) dx \quad f(x) = (6-x)^{1/2} \quad f'(x) = \frac{1}{2} (6-x)^{-1/2} (6-x)' = -\frac{1}{2\sqrt{6-x}}$$

$$dy = -\frac{1}{2\sqrt{6-x}} dx$$

$$\text{When } x=3 \quad dx=0.2 \quad dy = -\frac{1}{2\sqrt{3}} (0.2) = \boxed{-.0577350269}$$

$$\text{When } x=3 \quad dx=0.04 \quad dy = -\frac{1}{2\sqrt{3}} (0.04) = \boxed{-.0115470054}$$

$$\frac{dr}{dt} = 7 \text{ in/min}$$

$$V = \frac{4}{3} \pi r^3$$

Determine the rate at which the volume is changing with respect to time when $r=19$ in

$$\frac{dV}{dt} = \frac{4}{3} \pi (3r^2) \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi (19)^2 7 = \boxed{10,108\pi} \text{ in}^3/\text{min}$$

$$r=19$$

$$\frac{dr}{dt} = 7$$

Correct notation is very important here

$$\textcircled{6} \quad \lim_{x \rightarrow -\infty} \frac{5x+1}{-x-10} = \lim_{x \rightarrow -\infty} \frac{5}{-1} = \lim_{x \rightarrow -\infty} -5 = \boxed{-5}$$

this is $\frac{-\infty}{\infty}$ form, we apply
H rule

Correct notation is very important for limits

$$\textcircled{7} \quad \lim_{x \rightarrow 0} \frac{6e^x - 6 - 6x}{3x^2} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{6e^x - 6}{6x} = \lim_{x \rightarrow 0} \frac{6e^x}{6} = \frac{6e^0}{6} = \boxed{1}$$

$6e^0 - 6 - 6(0) = 0$
so we have $\frac{0}{0}$
and we use H rule

$6e^0 - 6 = 0$
we have $\frac{0}{0}$
we use H rule again

$$f(x) = x^4 - 6x^3$$

$$f'(x) = 4x^3 - 18x^2 = 0$$

$$x^2(4x - 18) = 0$$

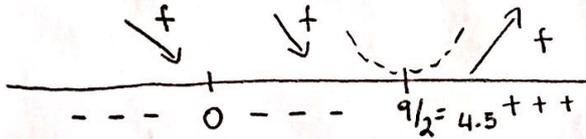
$$x = 0$$

$$x = \frac{18}{4} = \frac{9}{2}$$

critical values

$$\boxed{0, \frac{9}{2}}$$

For parts B-c-D-E we use a sign chart for $f'(x) = 4x^3 - 18x^2$



$$f'(-1) =$$

$$4(-1)^3 - 18(-1)^2$$

$$= -4 - 18 = -22 < 0$$

$$f'(1) = 4 - 18 =$$

$$-14 < 0$$

$$f'(5) = 4(5)^3 - 18(25) =$$

$$500 - 450 = 50 > 0$$

f is increasing in

$$\boxed{[\frac{9}{2}, \infty)}$$

f is decreasing in

$$\boxed{(-\infty, \frac{9}{2})}$$

Local maximums

NONE

Local minimums at $x =$

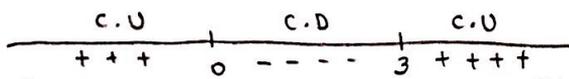
$$\boxed{\frac{9}{2}}$$

(f changes from decreasing to increasing)

For parts F-G-H we use a sign chart for $f''(x) = 12x^2 - 36x$

$$12x^2 - 36x = 0$$

$$12x(x-3) = 0 \quad x=0 \quad x=3$$



$$f''(-1) =$$

$$12(-1)^2 - 36(-1) =$$

$$12 + 36 = 48 > 0$$

$$f''(1) =$$

$$12 - 36 = -24 < 0$$

$$f''(4) = 12(16) - 36(4) =$$

$$192 - 144 = 48 > 0$$

f is concave up in

$$\boxed{(-\infty, 0) \cup (3, \infty)}$$

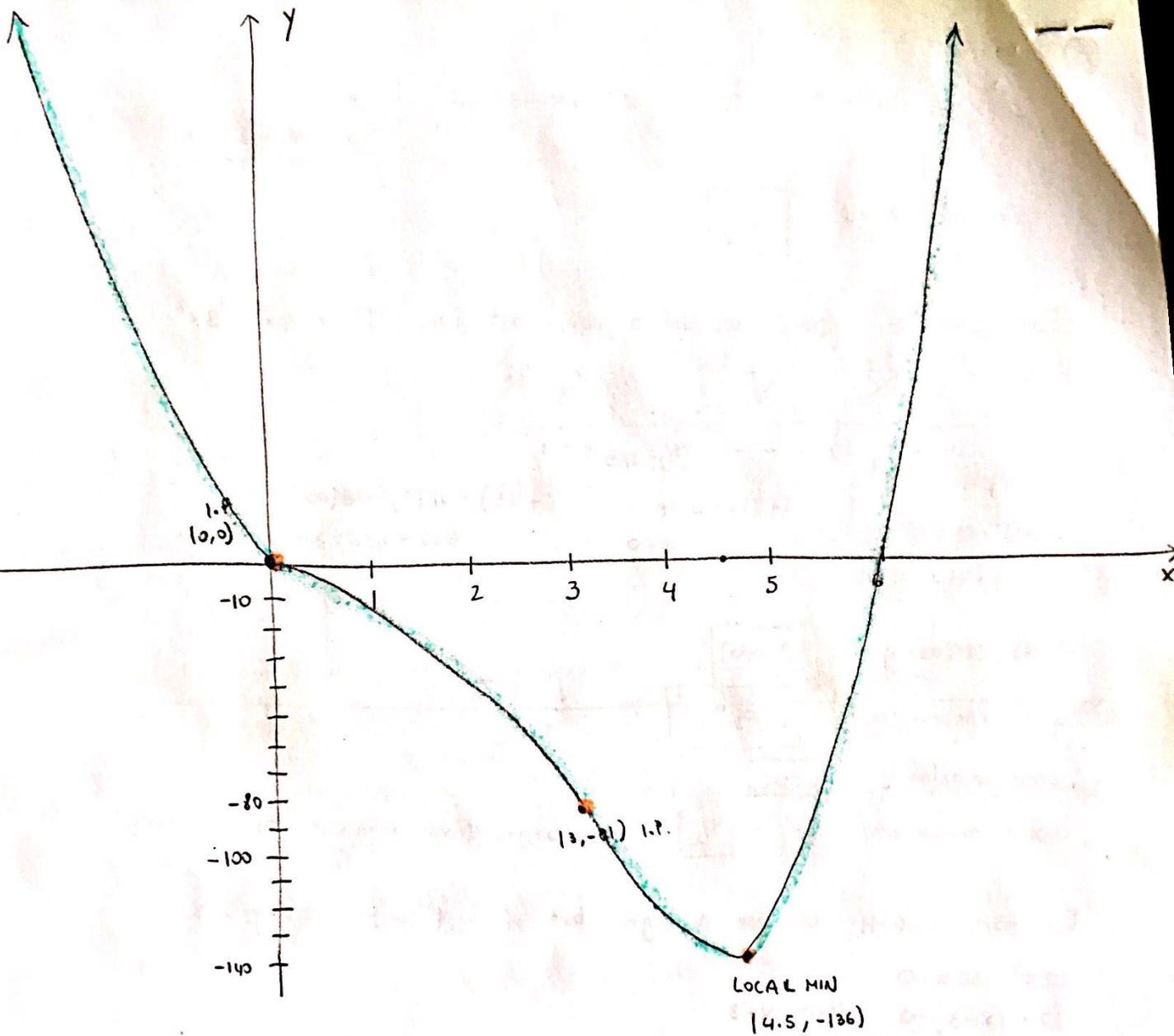
f is concave down in

$$\boxed{(0, 3)}$$

f has inflection points at $x=0$ and $x=3$ because f changes concavity there

x-values of inflection points

$$\boxed{0, 3}$$



We start by plotting the points we have

$$x=0 \quad f(0)=0 \quad (0,0) \text{ inflection point}$$

$$x = 9/2 = 4.5 \quad f(4.5) = (4.5)^4 - 6(4.5)^3 = -136.6875 \approx -136 \quad (4.5, -136) \text{ local min}$$

$$x=3 \quad f(3) = 3^4 - 6(3^3) = -81 \quad (3, -81) \text{ inflection point}$$

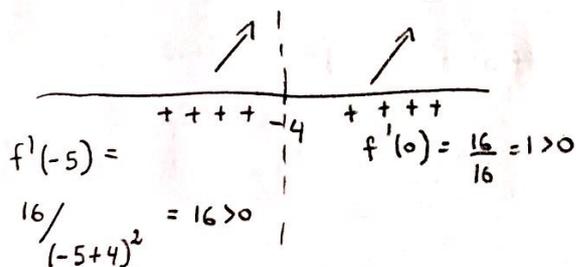
$$f(x) = \frac{3x-4}{x+4} \quad \text{domain } (-\infty, -4) \cup (-4, \infty)$$

$$\text{critical values } f'(x) = \frac{(3x-4)'(x+4) - (3x-4)(x+4)'}{(x+4)^2} = \frac{3(x+4) - (3x-4)(1)}{(x+4)^2}$$

$$= \frac{\cancel{3x} + 12 - \cancel{3x} + 4}{(x+4)^2} = \frac{16}{(x+4)^2}$$

$16 = 0$ never there are no critical values **NONE**

For B-C-D-E we need a sign chart for $f'(x)$



(we use -4 in the chart because it is a zero of the denominator)

f is increasing in $\boxed{(-\infty, -4) \cup (-4, \infty)}$

f is decreasing in $\boxed{\{ \}$ (never)

There are no critical values, so there are no local maxima and no local minima

Local maxima **none**

Local minima **none**

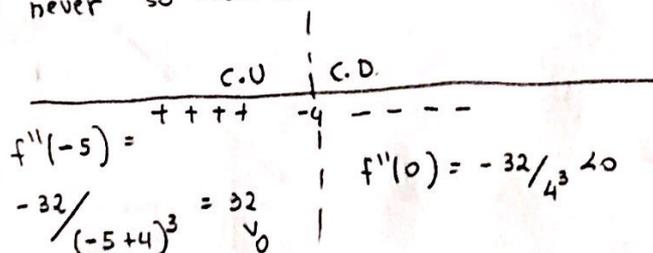
For F-G-H we need a sign chart for $f''(x)$

$$f'(x) = \frac{16}{(x+4)^2} = 16(x+4)^{-2}$$

$$f''(x) = -32(x+4)^{-3}(x+4)' = \frac{-32}{(x+4)^3}$$

(You can also use the chain rule)

$-32 = 0$ never so there are no I.P.



f is concave up in $\boxed{(-\infty, -4)}$

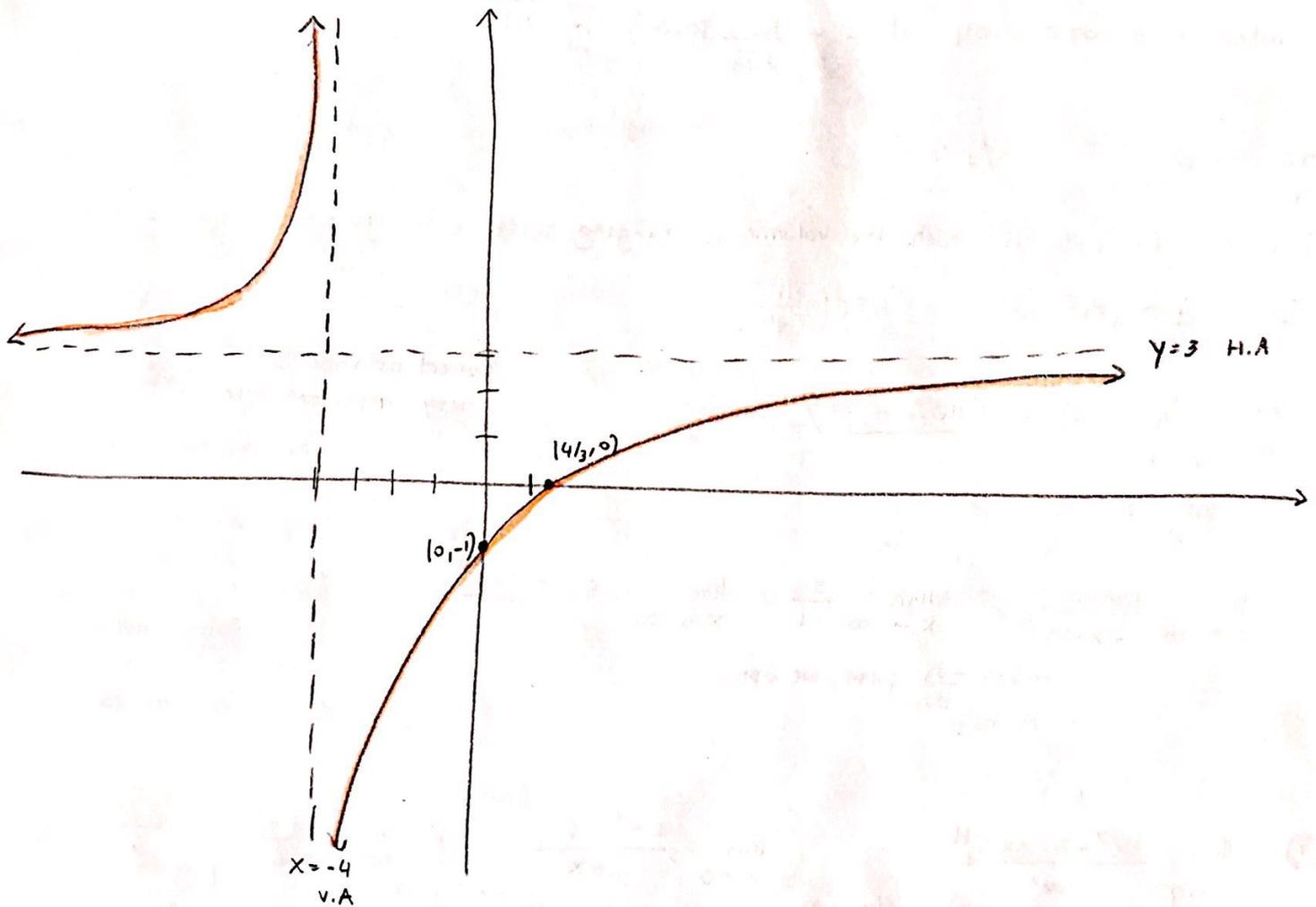
Inflection points **NONE**

f is concave down in $\boxed{(-4, \infty)}$

H.A. $\lim_{x \rightarrow \pm\infty} \frac{3x-4}{x+4} = \lim_{x \rightarrow \pm\infty} \frac{3}{1} = 3$ $y=3$ is H.A.

V.A. $x+4=0$ $x=-4$

$\lim_{x \rightarrow -4} \frac{3x-4}{x+4} = \frac{-16}{0}$ infinite limit so $x=-4$ is V.A.



$f(x) = \frac{3x-4}{x+4}$

x-int. $3x-4=0$ $(\frac{4}{3}, 0)$
 $x = \frac{4}{3}$

y-int $f(0) = -1$ $(0, -1)$