RELATED RATES

rate of change of x with respect to t

<u>Ex1</u> If $y = x^3 + 2x$ and $\frac{dx}{dt} = 5$, find $\frac{dy}{dt}$ when $x = 2$

relation

Here $x = x(t)$ $\quad$ t = time

$y = y(t)$ $\quad$ independent variable

(1) $y = x^3 + 2x$

(2) derivative wrt t

$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} + 2 \frac{dx}{dt}$$

$\frac{dy}{dt}$ is "related" to $\frac{dx}{dt}$

(3) plug in $\frac{dx}{dt} = 5$ $\quad x = 2$

$$\frac{dy}{dt} = 3(4)(5) + 2(5) = 60 + 10 = \boxed{70}$$

$\boxed{\text{Ex 2}}$ Suppose oil spills from a tank and spreads in a circular pattern. If the radius of the oil spill increases at a rate of 1 METER/SEC, How fast is the <u>area</u> of the spill increasing when the radius is 30 METERS?

derivatives $\Leftrightarrow$ RATE of change

$R = R(t)$ radius at time t

$A = A(t)$ area at time t

(1) $A = \pi R^2$

Notation is IMPORTANT

(2) $\frac{dA}{dt} = \pi \, 2R \frac{dR}{dt}$ $\quad$ related rates

(3) $\frac{dA}{dt} = \pi (60)(1)$

$\frac{dR}{dt} = 1$ METER/SEC

$R = 30$ METERS

$= 60\pi$ METERS^2/SEC

units of A / units of t $\quad \approx 188.5$ M^2/SEC

EX 3 The radius of a sphere is increasing at a rate of 4 MM/SEC. How fast is the volume increasing when the diameter is 80 MM?

$R = R(t)$ at time t

$V = v(t)$

$D = \text{diameter} = 2R$

① $V = \underbrace{\frac{4}{3}\pi}_{\text{constant}} R^3$ volume of a sphere

② $\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}R^2)\frac{dR}{dt} = 4\pi \overset{\text{RADIUS}}{R}^2 \frac{dR}{dt}$

③ $\frac{dV}{dt} = 4\pi(40)^2(4) = 16\pi(1600) = 25,600\pi \ MM^3/SEC$

exact

$R = \frac{D}{2}$ $\begin{cases} D = 80 \text{ MM} \\ R = 40 \text{ MM} \end{cases}$ $\frac{dR}{dt} = 4$

$\approx 80424.77 \ MM^3/SEC$

approx.

EX 4 A cylindrical tank with radius 5 Meters is being filled with water at a rate of $3 \ M^3/MIN$

How fast is the height of the water increasing?

① $V = \pi R^2 h = 25\pi h$

$R = 5$

↓

volume of the cylinder $v(t)$ $R = 5$

$h(t)$

$V = 25\pi h$

② $\frac{dV}{dt} = 25\pi \frac{dh}{dt}$ ← we need to find this $3M^3/MIN = \frac{dV}{dt}$

Given

③ $3 = 25\pi \boxed{\frac{dh}{dt}}$

$$\frac{3}{25\pi} = \frac{dh}{dt} \qquad \frac{dh}{dt} = \frac{3}{25\pi} \text{ METERS/MINUTE}$$

$\approx .377$ METERS/MINUTE

AREA = SQUARE UNITS
VOLUME = CUBIC UNITS
HEIGHT = UNITS

$\boxed{\text{Ex 5}}$ If a snowball melts so that its <u>surface area</u> decreases at a rate of $1\ \text{cm}^2/\text{min}$, find the rate at which the diameter decreases when the diameter is 10 cm.

DECREASE ⟷ NEGATIVE DERIVATIVE

① $A = 4\pi R^2$ surface area

$A = A(t)$
$R = R(t)$
$D = D(t) = 2R$

② $\frac{dA}{dt} = 4\pi\,(2R)\,\frac{dR}{dt} = 8\pi R \frac{dR}{dt}$ → find

③ $-1 = 8\pi(5)\ dR/dt$

$\frac{dA}{dt} = -1 \qquad R = \frac{10}{2} = 5$

$\frac{-1}{40\pi} = \frac{40\cancel{\pi}}{\cancel{40\pi}} \frac{dR}{dt} \qquad \frac{dR}{dt} = -\frac{1}{40\pi}$

④ Diameter $\quad D = 2R \rightarrow \frac{dD}{dt} = 2\frac{dR}{dt}$

$g = 2f$
$g' = 2f'$

$$\frac{dD}{dt} = 2\left(-\frac{1}{40\pi}\right) = \boxed{-\frac{1}{20\pi}\ \text{CM}/\text{MIN}}$$

$$\approx -0.0159\ \text{CM}/\text{MIN}$$

Ex 6 The radius of a cone is increasing at a rate of 3 inches/sec and the height of the cone is 3 times the radius.

Find the rate of change for the volume of the cone when the radius is 7 inches.

① $V = \frac{1}{3}\pi R^2 h$ $\quad V(t)$ $\quad \frac{dR}{dt} = 3$

$R(t)$

$\cancel{h(t)}$

$h = 3R$

$= \frac{1}{\cancel{3}}\pi R^2 (\cancel{3}R)$

$h = 3R$

always

$V = \pi R^3$

② $\frac{dV}{dt} = \pi\, 3R^2 \frac{dR}{dt}$

③ $\frac{dV}{dt} = \pi\, \overbrace{3(7)^2\, 3}^{441} = \boxed{441\pi\ \text{in}^3/\text{sec}}$

$R = 7 \quad \frac{dR}{dt}$

$\approx 1385.44\ \text{in}^3/\text{sec}$