① $y = (x^5+3)^4$

$y' = 4(x^5+3)^3 (x^5+3)' = 4(x^5+3)^3 (5x^4) = \boxed{20x^4 (x^5+3)^3}$

② $f(x) = f(g(x))$ $\quad g(3)=6$ $\quad f'(3)=-6$ $\quad$ Find $F'(3)$

$g'(3)=4$ $\quad f'(6)=7$

$F'(x) = f'(g(x))\, g'(x)$

$F'(3) = f'(g(3))\, g'(3) = f'(6)\, g'(3) = 7 \cdot 4 = \boxed{28}$

③ $\frac{d}{dx} \cos(u)$

a) $u(x) = 6-x^2$ $\quad \frac{d}{dx} \cos(6-x^2) = -\sin(6-x^2)(-2x) = \boxed{2x \sin(6-x^2)}$

b) $u(x) = x^{-1}$ $\quad \frac{d}{dx} \cos(x^{-1}) = -\sin(x^{-1})(x^{-1})' = -\sin(x^{-1})(-1x^{-2}) = \boxed{\frac{\sin(x^{-1})}{x^2}}$

c) $u(x) = \tan x$ $\quad \frac{d}{dx} \cos(\tan x) = \boxed{-\sin(\tan x) * \sec^2 x}$

④ $f(x) = e^{5x}(x^2+6^x)$ $\quad$ product rule

$f'(x) = (e^{5x})'(x^2+6^x) + e^{5x}(x^2+6^x)'$

$= \boxed{5e^{5x}(x^2+6^x) + e^{5x}(2x+6^x \ln 6)}$

⑤ $y = x^{9\tan(x)}$

$\ln y = \ln(x^{9\tan(x)})$

$\ln y = 9\tan(x) \ln x$

$(\ln y)' = (9\tan(x)\ln x)'$ $\quad$ product rule

$\frac{y'}{y} = (9\tan(x))' \ln x + 9\tan(x)(\ln x)'$

$\frac{y'}{y} = 9\sec^2(x) \ln x + 9\tan x \cdot \frac{1}{x}$

$y' = y\left[9\sec^2(x)\ln x + 9\frac{\tan x}{x}\right]$

$\boxed{y' = x^{9\tan(x)}\left[9\sec^2(x)\ln x + \frac{9\tan x}{x}\right]}$

⑥ $y = \frac{\ln(8x)}{8x}$ quotient rule

$$y' = \frac{(\ln(8x))' 8x - \ln(8x)(8x)'}{(8x)^2} = \frac{\frac{8}{8x} \cdot 8x - \ln(8x) \cdot 8}{64x^2} = \frac{8[1 - \ln(8x)]}{64x^2} =$$

$$\boxed{\frac{1 - \ln(8x)}{8x^2}}$$

⑦ $5xy^3 + 9xy = 58$ slope at $(1,2)$

$(5xy^3)' + (9xy)' = (58)'$

product rule

$(5x)' y^3 + (5x)(y^3)' + (9x)' y + (9x) y' = 0$

$5y^3 + 5x(3y^2 y') + 9y + 9xy' = 0$ solve for y'

$5y^3 + 15xy^2 y' + 9y + 9xy' = 0$

$-5y^3 \qquad -9y$

$15xy^2 y' + 9xy' = -5y^3 - 9y$

$y'(15xy^2 + 9x) = -5y^3 - 9y \qquad y' = \frac{-5y^3 - 9y}{15xy^2 + 9x}$

at $(1,2)$ $y' = \frac{-5 \cdot 2^3 - 9 \cdot 2}{15 \cdot 4 + 9} = \frac{-40 - 18}{60 + 9} = \boxed{\frac{-58}{69}}$

⑧ $y = 4x \cos x$ tangent line at $(\pi, -4\pi)$

$y' = (4x)' \cos x + 4x(\cos x)' = 4\cos x + 4x(-\sin x) = 4\cos x - 4x\sin x$

product rule

slope at $x = \pi$ $y'(\pi) = 4\cos(\pi) - 4\pi \underbrace{\sin(\pi)}_{0} = 4(-1) = -4$

$y = -4x + b$ plug in $(\pi, -4\pi)$ to find b

$-4\pi = -4\pi + b \quad b = 0$

$y = -4x$ $\boxed{M = -4 \quad b = 0}$

⑨ $f(x) = \tan^{-1}(x^3)$ $f'(x) = \underbrace{\frac{(x^3)'}{1 + (x^3)^2}}_{\text{chain rule}} = \boxed{\frac{3x^2}{1 + x^6}}$

⑩ $h(t) = 100 + 45t - 5t^2$ after 6 seconds $v(t) = h'(t) = 45 - 10t$

a) $h(6) = 100 + 45(6) - 5(36) = 100 + 270 - 180 = \boxed{190}$ m

b) $v(6) = 45 - 10(6) = 45 - 60 = \boxed{-15}$ m/s