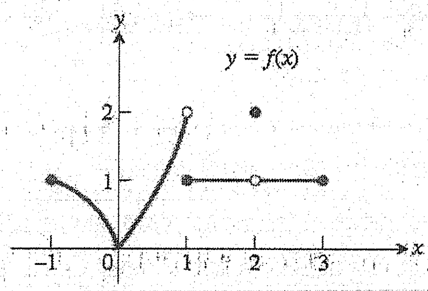
New York City College of Technology
MAT 1475 - Prof. Ghezzi
Exam 1 - Version A - Total Points: 105

NAME: Key

Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner.**

---

1. Use the graph below to answer the following questions.



a) (3 points-no partial credit) $f(2) =$ 2

b) (3 points-no partial credit) $\lim_{x\to 2^-} f(x) =$ 1

c) (3 points-no partial credit) $\lim_{x\to 2^+} f(x) =$ 1

d) (3 points-no partial credit) $\lim_{x\to 2} f(x) =$ 1

e) (3 points-no partial credit) Circle the most accurate statement:

i. $f(x)$ is continuous at $x = 2$.

ii. $f(x)$ is not continuous at $x = 2$ because $f(2)$ is undefined.

iii. $f(x)$ is not continuous at $x = 2$ because $\lim_{x\to 2} f(x)$ does not exist.

iv. $f(x)$ is not continuous at $x = 2$ because $f(2)$ and $\lim_{x\to 2} f(x)$ exist, but are not equal.

f) (3 points-no partial credit) $\lim_{x\to 0} f(x) =$ 0

2. (3 points-no partial credit) Given that $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} g(x) = -2$, find $\lim_{x\to 3}(2g(x) - f(x)g(x))$.

$$= 2\lim_{x\to 3} g(x) - \lim_{x\to 3} f(x) \lim_{x\to 3} g(x) = 2(-2) - 4(-2) = -4+8 = \boxed{4}$$

3. (4 points each) Evaluate the following limits algebraically. **Show your work and make sure you use correct notation.** If the limit is not a number write "does not exist".

a) $\lim_{x \to 2} x^3 - 4x$ $= 8 - 8 = \boxed{0}$

b) $\lim_{x \to 0} \frac{x+3}{x^2-9}$ $= \frac{0+3}{0-9} = \boxed{-\frac{1}{3}}$

c) $\lim_{x \to -3} \frac{x+3}{x^2-9}$ $= \lim_{x \to -3} \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \lim_{x \to -3} \frac{1}{x-3} = \boxed{-\frac{1}{6}}$

$\frac{0}{0}$ factor

d) $\lim_{x \to 3} \frac{x+3}{x^2-9}$ $= \frac{6}{0}$ $\boxed{\text{DNE}}$

e) $\lim_{x \to 7} \frac{x-7}{x+7}$ $= \frac{0}{14} = \boxed{0}$

f) $\lim_{x \to -8} 13$ $= \boxed{13}$

g) $\lim_{x \to 6} \frac{x^2-36}{x^2-5x-6}$ $= \lim_{x \to 6} \frac{(x+6)\cancel{(x-6)}}{\cancel{(x-6)}(x+1)} = \lim_{x \to 6} \frac{x+6}{x+1} = \boxed{\frac{12}{7}}$

$\frac{0}{0}$ factor

4. a) (12 points) Find the derivative of $f(x) = 3x^2 - 4x + 8$ using the **definition** (the formula with limit).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - 4(x+h) + 8 - (3x^2 - 4x + 8)}{h} =$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 4x - 4h + 8 - 3x^2 + 4x - 8}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{4x} - 4h \cancel{+8} - \cancel{3x^2} + \cancel{4x} \cancel{-8}}{h} = \lim_{h \to 0} \frac{6xh + 3h^2 - 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 4)}{\cancel{h}} \underset{h=0}{=} \boxed{6x - 4}$$

b) (5 points) Find the equation of the tangent line to $f(x) = 3x^2 - 4x + 8$ at the point $(1, 7)$.

Slope $= f'(1) = 6(1) - 4 = 2$

$y = 2x + b$ plug in $(1,7)$ to find b

$7 = 2 + b$ $b = 5$

$\boxed{y = 2x + 5}$

5. (4 points) Given that $f(3) = 2$, $f'(3) = -5$, $g(3) = -4$, $g'(3) = 6$, find $h'(3)$ for $h(x) = f(x)g(x)$.

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(3) = f'(3)g(3) + f(3)g'(3) = (-5)(-4) + 2(6) = 20 + 12 = \boxed{32}$

6. Find the derivatives of the following functions:

(a) (2 points-no partial credit) $f(x) = 4x^6$. $f'(x) = 24x^5$

(b) (2 points-no partial credit) $f(x) = 4^6$. $f'(0) = 0$

(c) (3 points-no partial credit) $f(x) = -8x + 17$. $f'(x) = -8$

(d) (4 points) $f(x) = \sqrt[4]{x}$. Write your final answer in radical form.

$f(x) = x^{1/4}$ so $f'(x) = \frac{1}{4} x^{1/4 - 1} = \frac{1}{4} x^{-3/4} = \frac{1}{4x^{3/4}} = \boxed{\frac{1}{4\sqrt[4]{x^3}}}$

(e) (4 points) $f(x) = \frac{1}{x^3}$. Write your final answer using positive exponents.

$f(x) = x^{-3}$ so $f'(x) = -3x^{-4} = \boxed{\frac{-3}{x^4}}$

(f) (5 points) $f(x) = x^3(6x^5 - 7)$. (Simplify your answer.)

$f'(x) \underset{\text{product rule}}{=} (x^3)'(6x^5-7) + x^3(6x^5-7)' = 3x^2(6x^5-7) + x^3(30x^4) =$

$18x^7 - 21x^2 + 30x^7 = \boxed{48x^7 - 21x^2}$

Other solution: $f(x) = 6x^8 - 7x^3$ so $f'(x) = 48x^7 - 21x^2$

7. (10 points) Find the derivative of $f(x) = \dfrac{x^2+6}{x^3-3x}$. Show all steps and simplify your answer.

Quotient rule $f'(x) = \dfrac{(x^2+6)'(x^3-3x) - (x^2+6)(x^3-3x)'}{(x^3-3x)^2} = \dfrac{2x(x^3-3x) - (x^2+6)(3x^2-3)}{(x^3-3x)^2} \underset{\text{algebra}}{=}$

$= \dfrac{2x^4 - 6x^2 - [3x^4 - 3x^2 + 18x^2 - 18]}{(x^3-3x)^2} = \dfrac{2x^4 - 6x^2 - 3x^4 + 3x^2 - 18x^2 + 18}{(x^3-3x)^2} =$

$\boxed{\dfrac{-x^4 - 21x^2 + 18}{(x^3-3x)^2}}$

---

8. (5 points extracredit)

$$f(x) = \begin{cases} x^2 & x < 3 \\ -2x + C & x > 3 \end{cases}$$

For which value of C does $\lim_{x\to 3} f(x)$ exist? Show your work.

We need $\lim_{x\to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\lim_{x\to 3^-} f(x) = 9$ $\quad \lim_{x\to 3^+} f(x) = -6 + C$ $\quad 9 = -6 + C$ $\quad \boxed{C = 15}$

New York City College of Technology

MAT 1475 - Prof. Ghezzi

Exam 1 - Version B - Total Points: 105

NAME: Key

Instructions: Write your solutions in the space provided after each question. You may use the back of each page for any scratch work that you need to do. To receive (partial) credit you must **show all your work in a clear and organized manner.**

---

1. (4 points each) Evaluate the following limits algebraically. **Show your work and make sure you use correct notation.** If the limit is not a number write "does not exist".

a) $\lim_{x\to 3} x^2+5x+1$ $= 9+15+1 = \boxed{25}$

b) $\lim_{x\to 0} \frac{x-2}{x^2-4}$ $= \frac{0-2}{0-4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

c) $\lim_{x\to -2} \frac{x-2}{x^2-4}$ $= \frac{-2-2}{(-2)^2-4} = \frac{-4}{0}$ $\boxed{\text{DNE}}$

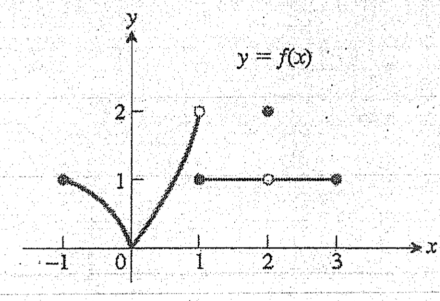
d) $\lim_{x\to 2} \frac{x-2}{x^2-4}$ $\underset{\frac{0}{0}}{=}$ $\underset{\text{FACTOR}}{\lim_{x\to 2}} \frac{(\cancel{x-2})}{(\cancel{x-2})(x+2)} = \lim_{x\to 2} \frac{1}{x+2} = \boxed{\frac{1}{4}}$

e) $\lim_{x\to 6} \frac{x-6}{x+6}$ $= \frac{6-6}{6+6} = \frac{0}{12} = \boxed{0}$

f) $\lim_{x\to 11} 24$ $= \boxed{24}$

g) $\lim_{x\to -5} \frac{x^2-25}{x^2+4x-5}$ $\underset{\frac{0}{0}\text{ factor}}{=}$ $\lim_{x\to -5} \frac{(x-5)(\cancel{x+5})}{(x-1)(\cancel{x+5})} = \lim_{x\to -5} \frac{(x-5)}{(x-1)} = \frac{-10}{-6} = \boxed{\frac{5}{3}}$

2. Use the graph below to answer the following questions.



a) (3 points-no partial credit) $f(1) =$ 1

b) (3 points-no partial credit) $\lim_{x \to 1^-} f(x) =$ 2

c) (3 points-no partial credit) $\lim_{x \to 1^+} f(x) =$ 1

d) (3 points-no partial credit) $\lim_{x \to 1} f(x) =$ DNE

e) (3 points-no partial credit) Circle the most accurate statement:

i. $f(x)$ is continuous at $x = 1$.

ii. $f(x)$ is not continuous at $x = 1$ because $f(1)$ is undefined.

iii. $f(x)$ is not continuous at $x = 1$ because $\lim_{x \to 1} f(x)$ does not exist.

iv. $f(x)$ is not continuous at $x = 1$ because $f(1)$ and $\lim_{x \to 1} f(x)$ exist, but are not equal.

f) (3 points-no partial credit) $\lim_{x \to 0} f(x) =$ 0

3. (3 points-no partial credit) Given that $\lim_{x \to 5} f(x) = 4$ and $\lim_{x \to 5} g(x) = -3$, find $\lim_{x \to 5}(2f(x) - f(x)g(x))$.

$= 2 \lim_{x \to 5} f(x) - \lim_{x \to 5} f(x) \lim_{x \to 5} g(x) = 2(4) - 4(-3) = 8 + 12 = \boxed{20}$

4. a) (12 points) Find the derivative of $f(x) = 2x^2 - 3x + 6$ using the **definition** (the formula with limit).

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) + 6 - (2x^2-3x+6)}{h} =$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - \cancel{3x} - 3h + \cancel{6} - 2x^2 + \cancel{3x} - \cancel{6}}{h} = \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 3h - \cancel{2x^2}}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x+2h-3)}{\cancel{h}} = \boxed{4x-3}$$

b) (5 points) Find the equation of the tangent line to $f(x) = 2x^2 - 3x + 6$ at the point $(1, 5)$.

Slope : $f'(1) = 1$

$y = x + b$ plug in $(1,5)$

$5 = 1 + b$ $b = 4$

$\boxed{y = x + 4}$

5. (4 points) Given that $f(4) = -2$, $f'(4) = 5$, $g(4) = -4$, $g'(4) = 3$, find $h'(4)$ for $h(x) = f(x)g(x)$.

$h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

$h'(4) = f'(4)\,g(4) + f(4)\,g'(4) = 5(-4) + (-2)(3) = -20 - 6 = \boxed{-26}$

6. Find the derivatives of the following functions:

(a) (2 points-no partial credit) $f(x) = 6x^7$. $f'(x) = 42x^6$

(b) (2 points-no partial credit) $f(x) = 6^7$. $f'(x) = 0$

(c) (3 points-no partial credit) $f(x) = -9x + 35$. $f'(x) = -9$

(d) (4 points) $f(x) = \sqrt[5]{x}$. Write your final answer in radical form.

$f(x) = x^{1/5}$ so $f'(x) = \frac{1}{5} x^{1/5 - 1} = \frac{1}{5} x^{-4/5} = \frac{1}{5x^{4/5}} = \boxed{\frac{1}{5\sqrt[5]{x^4}}}$

(e) (4 points) $f(x) = \frac{1}{x^6}$. Write your final answer using positive exponents.

$f(x) = x^{-6}$ so $f'(x) = -6x^{-7} = \boxed{\frac{-6}{x^7}}$

(f) (5 points) $f(x) = x^8(3x^4 - 2)$. (Simplify your answer.)

$f'(x) =$ product rule $(x^8)'(3x^4-2) + x^8(3x^4-2)' = 8x^7(3x^4-2) + x^8(12x^3) = 24x^{11} - 16x^7 + 12x^{11} = \boxed{36x^{11} - 16x^7}$

other solution: $f(x) = 3x^{12} - 2x^8$ so $f'(x) = 36x^{11} - 16x^7$

7. (10 points) Find the derivative of $f(x) = \frac{x^3 + 8}{x^2 - 4x}$. Show all steps and simplify your answer.

quotient rule

$$f'(x) = \frac{(x^3+8)'(x^2-4x) - (x^3+8)(x^2-4x)'}{(x^2-4x)^2} = \frac{(3x^2)(x^2-4x) - (x^3+8)(2x-4)}{(x^2-4x)^2} \overset{\text{algebra}}{=}$$

$$\frac{3x^4 - 12x^3 - (2x^4 - 4x^3 + 16x - 32)}{(x^2-4x)^2} = \frac{3x^4 - 12x^3 - 2x^4 + 4x^3 - 16x + 32}{(x^2-4x)^2}$$

$$= \boxed{\frac{x^4 - 8x^3 - 16x + 32}{(x^2-4x)^2}}$$

8. (5 points extracredit)

$$f(x) = \begin{cases} 4x + C & x < 2 \\ 2x^3 - 5x & x > 2 \end{cases}$$

For which value of C does $\lim_{x\to 2} f(x)$ exist? Show your work.

We need $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\lim_{x\to 2^-} f(x) = 4(2) + C = 8 + C$ $\quad$ $\lim_{x\to 2^+} f(x) = 2(2^3) - 5(2) = 16 - 10 = 6$

$8 + C = 6$ $\quad$ $\boxed{C = -2}$