① a) -2; b) $= -3$; c) DNE; d) -4; e) Statement 3; f) 2; g) Statement 1

② $\lim_{x \to 2} (5f(x) - 2g(x)) = 5 \lim_{x \to 2} f(x) - 2 \lim_{x \to 2} g(x) = 5(4) - 2(-3) = \boxed{26}$

③ a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5(x+h)^2 - 2(x+h) + 4 - (5x^2 - 2x + 4)}{h} =$

$= \lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) - \cancel{2x} - 2h + \cancel{4} - 5x^2 + \cancel{2x} - \cancel{4}}{h} = \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 - 2h - \cancel{5x^2}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(10x + 5h - 2)}{\cancel{h}} = \lim_{h \to 0} (10x + 5h - 2) = \boxed{10x - 2}$

b) Slope is $f'(-1) = 10(-1) - 2 = -12$

$y = -12x + b$ we use $(-1, 11)$ to find b

$11 = -12(-1) + b$ $\quad 11 = 12 + b \quad b = -1$ $\quad \boxed{y = -12x - 1}$

④ a) $\boxed{6}$

b) $3(-2)^2 - 7(-2) - 4 = 12 + 14 - 4 = \boxed{22}$

c) $\lim_{x \to 3} \frac{x-3}{x-3} = \lim_{x \to 3} 1 = \boxed{1}$

$\frac{0}{0}$ (common factor)

d) $\lim_{x \to 3} \frac{x-3}{x+3} = \lim_{x \to 3} \frac{0}{6} = \boxed{0}$

e) $\lim_{x \to 3} \frac{x+3}{x-3} = \frac{6}{0}$ $\quad \boxed{\text{DNE}}$

f) $\lim_{x \to -3} \frac{x+3}{x-3} = \frac{0}{-6} = \boxed{0}$

⑤ a) $\lim_{x \to 2} \frac{2x^2 - 32}{x^2 - 2x - 8} = \frac{8 - 32}{4 - 4 - 8} = \frac{-24}{-8} = \boxed{3}$

b) $\lim_{x \to -2} \frac{2x^2 - 32}{x^2 - 2x - 8} = \frac{8 - 32}{4 + 4 - 8} = \frac{-24}{0} = \boxed{\text{DNE}}$

c) $\lim_{x \to 4} \frac{2x^2 - 32}{x^2 - 2x - 8} =$ $\quad \lim_{x \to 4} \frac{2(x+4)\cancel{(x-4)}}{\cancel{(x-4)}(x+2)} = \frac{16}{6} = \boxed{\frac{8}{3}}$

$\frac{0}{0}$ common factor

⑥ In order for f to be continuous at $x = 1$ we need $f(1) = \lim_{x \to 1} f(x)$

To compute $\lim_{x \to 1} f(x)$ we compute right and left limits

$\lim_{x \to 1^-} f(x) \underset{\text{formula 1}}{=} 1^3 - 1 = 0$

$\lim_{x \to 1^+} f(x) \underset{\text{formula 2}}{=} 1 - 2 = -1$

So $\lim_{x \to 1} f(x)$ does not exist. Therefore f is not continuous at $x = 1$.

7. We need $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$$\underbrace{\lim_{x\to 1^-} f(x) = 1^3 - 1 = 0}_{\text{formula 1}} \qquad \underbrace{\lim_{x\to 1^+} f(x) = 1 + C}_{\text{formula 2}}$$

$0 = 1 + C$ so $\boxed{C = -1}$

8. a) $f'(x) = \boxed{45x^8}$

b) $f'(x) = \boxed{0}$ (f is constant)

c) $f(x) = x^{2/3}$ so $f'(x) = \frac{2}{3}x^{2/3-1} = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} = \boxed{\frac{2}{3\sqrt[3]{x}}}$

d) $f'(x) = \boxed{1}$

e) $f(x) = x^{-8}$ so $f'(x) = -8x^{-8-1} = -8x^{-9} = \boxed{-\frac{8}{x^9}}$

f) $f(x) = x^4 + x^{-1/2} + x^{-4}$ so $f'(x) = 4x^3 - \frac{1}{2}x^{-1/2-1} - 4x^{-4-1} =$

$= 4x^3 - \frac{1}{2}x^{-3/2} - 4x^{-5} = \boxed{4x^3 - \frac{1}{2\sqrt{x^3}} - \frac{4}{x^5}}$

g) $f(x) = \frac{1}{5}x$ so $f'(x) = \boxed{\frac{1}{5}}$

h) $$f'(x) \underset{\text{quotient rule}}{=} \frac{(x^2-4)'(x^2-5) - (x^2-4)(x^2-5)'}{(x^2-5)^2} = \frac{2x(x^2-5) - (x^2-4)(2x)}{(x^2-5)^2} =$$

$$= \frac{\cancel{2x^3} - 10x - \cancel{2x^3} + 8x}{(x^2-5)^2} = \boxed{\frac{-2x}{(x^2-5)^2}}$$

i) $$f'(x) \underset{\text{product rule}}{=} (x^5)'(3x^4-2x) + x^5(3x^4-2x)' = 5x^4(3x^4-2x) + x^5(12x^3-2) =$$

$15x^8 - 10x^5 + 12x^8 - 2x^5 = \boxed{27x^8 - 12x^5}$

other solution: $f(x) = 3x^9 - 2x^6$ so $f'(x) = 27x^8 - 12x^5$

9. a) $h'(2) = 3f'(2) = \boxed{6}$

b) $h'(2) \underset{\text{product rule}}{=} f'(2)g(2) + f(2)g'(2) = 2(-3) + 5(4) = -6 + 20 = \boxed{14}$

c) $$h'(2) \underset{\text{quotient rule}}{=} \frac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2} = \frac{2(-3) - 5(4)}{(-3)^2} = \frac{-6-20}{9} = \boxed{\frac{-26}{9}}$$

d) $h'(2) = f'(2) - 2g'(2) + 0 = 2 - 2(4) = \boxed{-6}$