1. Evaluate the given limits

(a). $\lim\limits_{x\to\frac{\pi}{2}} \sin x\cos x = \sin\frac{\pi}{2}\cos\frac{\pi}{2} = (1)(0) = \boxed{0}$

(b). $\lim\limits_{x\to 0} \ln(1+x) = \ln(1+0) = \ln 1 = \boxed{0}$

(c). $\lim\limits_{x\to 0} (e^{2x}+1) = e^{2(0)}+1 = 1+1 = \boxed{2}$

(d). $\lim\limits_{x\to 0} \dfrac{x^2+2x}{x^2-2x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \dfrac{x(x+2)}{x(x-2)} = \lim\limits_{x\to 0} \dfrac{x+2}{x-2} = \dfrac{2}{-2} = \boxed{-1}$

(e). $\lim\limits_{x\to 1} \dfrac{x^2-2x+1}{2x^2-x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 1} \dfrac{(x-1)^2}{(2x+1)(x-1)} = \lim\limits_{x\to 1} \dfrac{x-1}{2x+1} = \dfrac{0}{3} = \boxed{0}$

(f). $\lim\limits_{h\to 0} \dfrac{(2+h)^2-4}{h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{h\to 0} \dfrac{4+4h+h^2-4}{h} = \lim\limits_{h\to 0} \dfrac{4h+h^2}{h}$

$= \lim\limits_{h\to 0} \dfrac{(4+h)\cdot h}{h} = \lim\limits_{h\to 0} 4+h = \boxed{4}$

(g). $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{x-9} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 9} \dfrac{(\sqrt{x}-3)\cdot(\sqrt{x}+3)}{(x-9)\cdot(\sqrt{x}+3)} = \lim\limits_{x\to 9} \dfrac{(x-9)}{(x-9)(\sqrt{x}+3)}$

$= \lim\limits_{x\to 9} \dfrac{1}{\sqrt{x}+3} = \boxed{\dfrac{1}{6}}$

(h). $\lim\limits_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 1} \dfrac{(\frac{1}{x}-1)\cdot x}{(x-1)\cdot x} = \lim\limits_{x\to 1} \dfrac{(1-x)}{(x-1)\cdot x} = \lim\limits_{x\to 1} \dfrac{-(x-1)}{(x-1)\cdot x}$

$= \lim\limits_{x\to 1} \dfrac{-1}{x} = \boxed{-1}$

2. Evaluate the limits of the piecewise defined functions and answer the question

$$g(x) = \begin{cases} 2x^2+5x-1, & \text{if } x<0 \\ \sin x & \text{if } x\geq 0 \end{cases}$$

(a) $\lim\limits_{x\to 0^-} g(x) = \lim\limits_{x\to 0^-} (2x^2+5x-1) = \boxed{-1}$

(b) $\lim\limits_{x\to 0^+} g(x) = \lim\limits_{x\to 0^+} \sin x = \sin 0 = \boxed{0}$

(c) $\lim\limits_{x\to 0} g(x) =$ DNE since $\lim\limits_{x\to 0^-} g(x) \neq \lim\limits_{x\to 0^+} g(x)$

(d). $g(0) = \sin 0 = \boxed{0}$

(e) $g(x)$ is not continuous at $x = 0$. It is a jump discontinuity

since $\lim_{x \to 0^-} g(x) \neq \lim_{x \to 0^+} g(x)$

3. Evaluate the limits of the piecewise defined functions and answer the question

$$f(x) = \begin{cases} x^2 & x < 2 \\ x+1 & x = 2 \\ -x^2 + 2x + 4 & x > 2 \end{cases}$$

(a). $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = \boxed{4}$

(b) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (-x^2 + 2x + 4) = -4 + 4 + 4 = \boxed{4}$

(c). $\lim_{x \to 2} f(x) = \boxed{4}$

(d) $f(2) = 2+1 = 3$

(e) $f(x)$ is discontinuous at $x = 2$. It is a removable discontinuity

since $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$ but it isn't equal to $f(2)$

4. Let $f(x) = -3x^2 + 2x - 1$

(a) Use the definition of the derivative to compute $f'(x)$

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{-3(x+h)^2 + 2(x+h) - 1 - (-3x^2 + 2x - 1)}{h} \\ &= \lim_{h \to 0} \frac{\cancel{-3x^2} - 6xh - 3h^2 + \cancel{2x} + 2h \cancel{-1} + \cancel{3x^2} - \cancel{2x} + \cancel{1}}{h} \\ &= \lim_{h \to 0} \frac{-6xh - 3h^2 + 2h}{h} \\ &= \lim_{h \to 0} \frac{(-6x - 3h + 2) \cdot \cancel{h}}{\cancel{h}} \\ &= \lim_{h \to 0} -6x - 3h + 2 = \boxed{-6x + 2} \end{aligned}$$

(b) $m = f'(1) = -6(1) + 2 = -4$

$(x_0, y_0) = (1, f(1)) = (1, -3(1)^2 + 2(1) - 1) = (1, -2)$

$\Rightarrow$ tangent line equation $y = -4(x-1) - 2$

$$\boxed{y = -4x + 2}$$

5. Use differential rules to find the derivative of the following functions

(a) $f(x) = x^{10} + 10$

$f'(x) = 10x^9 + 0 = \boxed{10x^9}$

(b) $f(x) = 4x^2 - 7x$

$f'(x) = 4 \cdot 2x - 7 \cdot 1 = \boxed{8x - 7}$

(c) $f(x) = x^4 + \frac{2}{x}$

rewrite $f(x) = x^4 + 2x^{-1}$

$f'(x) = 4 \cdot x^3 + 2 \cdot (-1)\, x^{-2} = \boxed{4x^3 - \frac{2}{x^2}}$

(d). $f(x) = (x+2)(2x^2 - 3)$

$$\begin{aligned} f'(x) &= (x+2)'(2x^2-3) + (x+2)(2x^2-3)' \\ &= 1 \cdot (2x^2-3) + (x+2)(2 \cdot 2x) \\ &= 2x^2 - 3 + 4x^2 + 8x \\ &= \boxed{6x^2 + 8x - 3} \end{aligned}$$

(e) $f(x) = \dfrac{x^3 + 2x^2 - 4}{3}$

rewrite $f(x) = \frac{1}{3} \cdot x^3 + \frac{2}{3} \cdot x^2 - \frac{4}{3}$

$f'(x) = \frac{1}{3} \cdot 3x^2 + \frac{2}{3} \cdot 2x - 0$

$= \boxed{x^2 + \frac{4}{3}x}$

(f). $f(x) = \dfrac{x^2 + 4}{x^2 - 4}$

$$f'(x) = \frac{(x^2+4)' \cdot (x^2-4) - (x^2+4)(x^2-4)'}{(x^2-4)^2} = \frac{2x(x^2-4) - (x^2+4) \cdot 2x}{(x^2-4)^2}$$

$$= \boxed{\frac{-16x}{(x^2-4)^2}}$$