Introduction

What follows is an instructor's guide to an activity designed for a class in Differential Calculus (generally, Calculus I). This lesson uses graphing technology to help students develop a conceptual understanding of tangent lines to graphs of functions, the slopes of those lines, and how the slope of a tangent line relates to the slopes of secant lines that are nearly tangent at that same point. This initial exploration can help give students a solid grounding in these key ideas before they go on to develop procedural fluency and speed by discovering shortcuts for differentiation. This activity emphasizes student voice, encourages students to identify and describe patterns, asks them to provide reasoning for their conclusions, and involves the whole class in gradually refining a definition of a tangent line based on information that arises over the course of the lesson.

Within this guide, you'll see a number of parenthetical notes set off from the main text, each seeking to explain something for the instructor that is not a part of the flow of the main lesson.

Logistical Note: These are practical details of the lesson like supplies, technology, or room setup, some of which will need to be arranged in advance.

Rabbit Hole Alert: These are moments where there's a possible direction of conversation we likely wish to avoid, most often because it would take us too far afield from the objectives of the lesson. In many rabbit holes, students may struggle because they don't yet have the tools they need to address the topic meaningfully, so they will not be able to reach useful conclusions on their own.

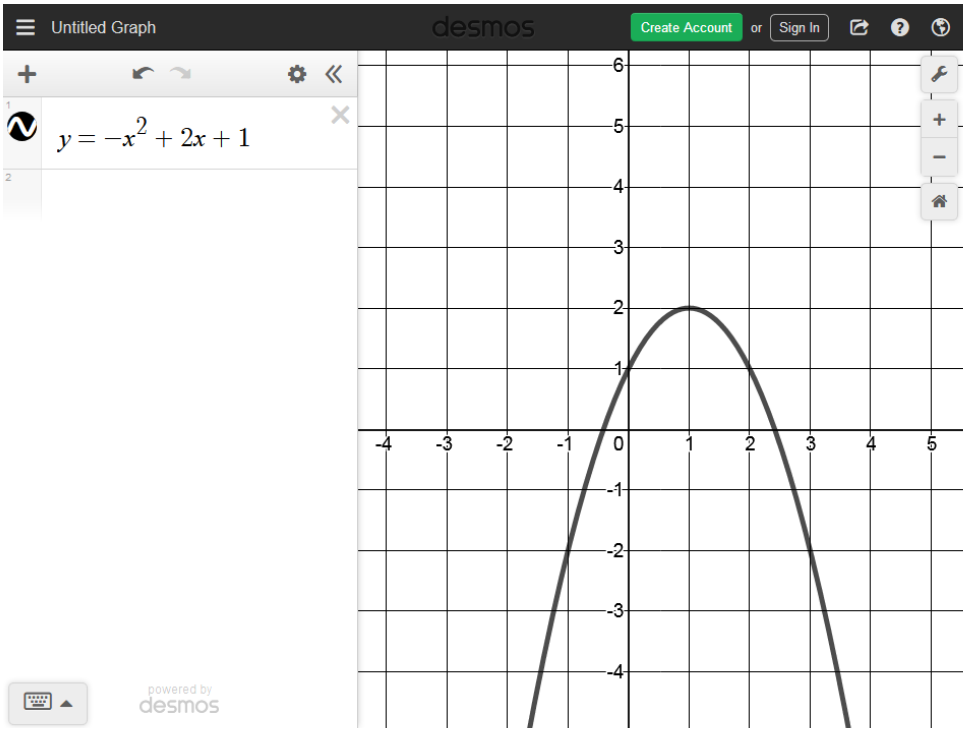
Spoiler Alert: In each of these moments, there is an idea we do not wish to give away to students too soon. Rather, through the flow of the lesson, we hope to have students reach these important conclusions on their own (with as little direction from the instructor as possible.)

Differentiation Note: In these moments, options are offered depending on the strengths or challenges of your group of students.

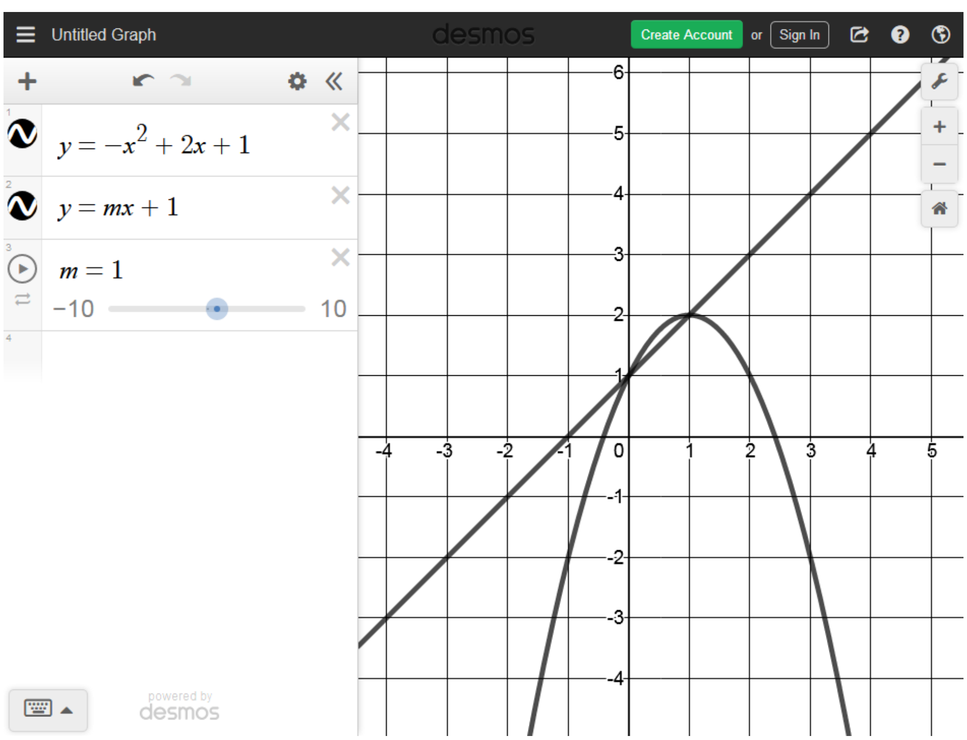
Lesson Design Note: These notes can provide the rationale for what we are suggesting at this moment in the lesson, or make some other comment or suggestion that does not fit neatly into one of the above categories.

Two Kinds of Lines

> Logistical Note: Well before class, practice using Desmos (https://www.desmos.com/calculator) to graph equations in the xy-plane. Try out some of the options for display and scale to be sure you are aware of any quirks of the interface that could inadvertently cause confusion for your students. For instance, as you zoom in or out on Desmos, the software automatically adjusts the scale of the grid lines and axis labels. This could be disorienting for any students who struggle with accurate graphing. One thing that can help with this is to uncheck "Minor Gridlines" and set the window for the x- and y-axes in the graph settings.
>
> Shortly before class, set up browser tabs with three pages open to the graphing calculator function on Desmos. On the first tab, graph the equation $y = -x^2 + 2x + 1$ with a window of about –5 to 5 for both x and y. See the model below.

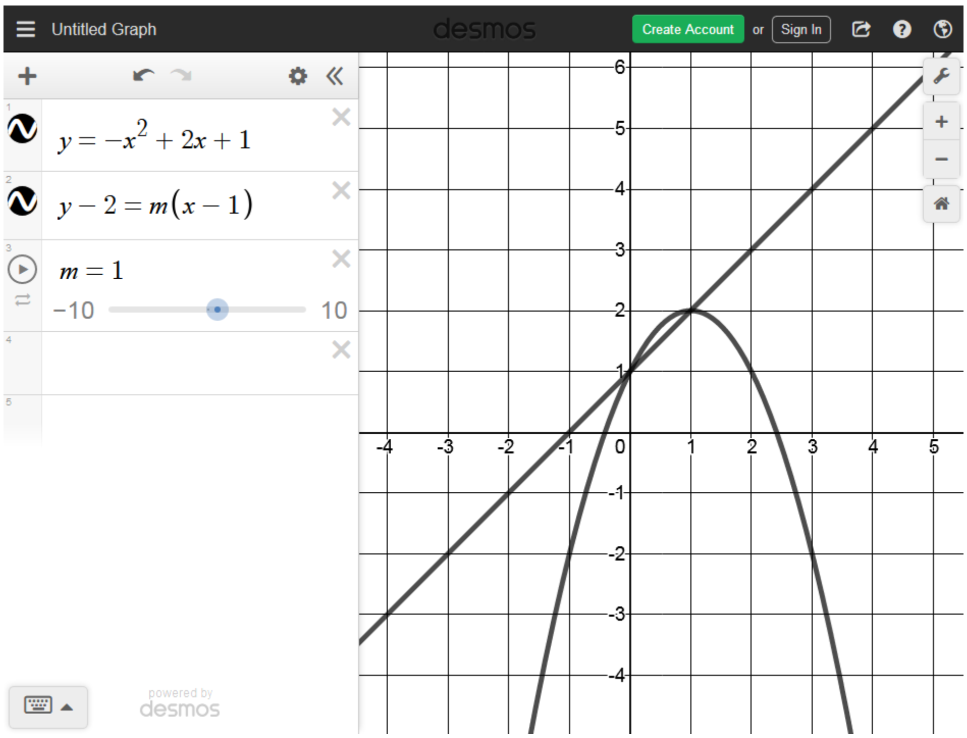


On the second tab, graph this same parabola, as well as a slider using the equation $y = mx + 1$. By clicking in the third row where it shows $m = 1$, you can set the range for the slider from –10 to 10 and the step at 0.5. See the model below.



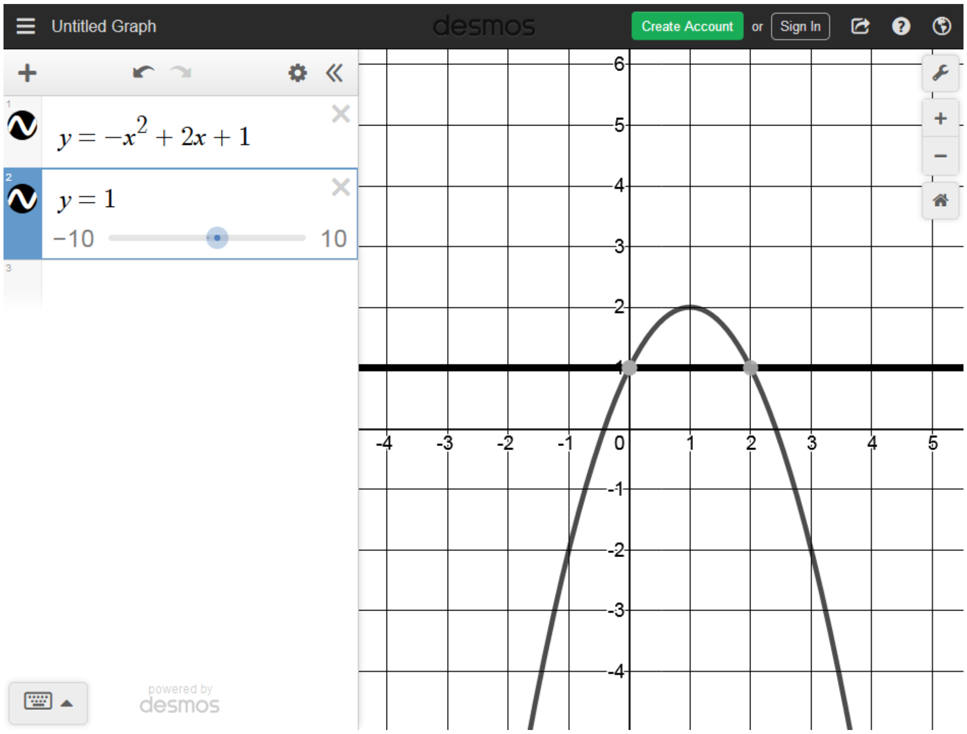
On the third tab, graph the same parabola and a slider using the equation $y-2=m(x-1)$. Uncheck "Minor Gridlines" in the settings menu for all three tabs. See the model below.

<u>Lesson design note</u> – The lines shown in the second and third tab look the same, but they have a different "hinge point" associated with the slider. This will help us do different things in the lesson.

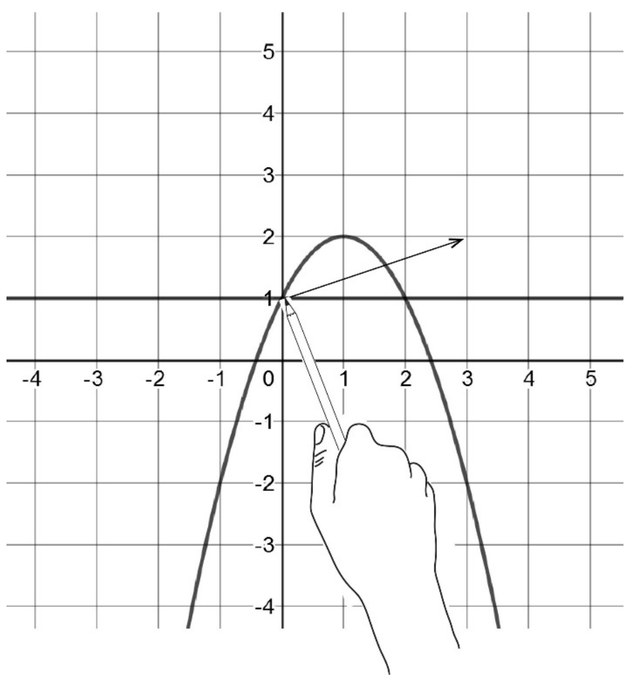


Display the graph in the first tab. Tell students that we're going to be thinking about the graph of the function $y=-x^2+2x+1$.

Graph the points (0,1) and (3,1) on the same Desmos tab and ask students to find the equation of the line that passes through these two points. After students identify the equation asZ $y=1$, ask how they know. Possible explanations could involve the common y-value of the two ordered pairs or realizing the slope would be 0 and using the slope-intercept or point-slope equation of a line. Enter the equation $y=1$ as a second function on Desmos, and have students confirm they found the correct line. See the model below.

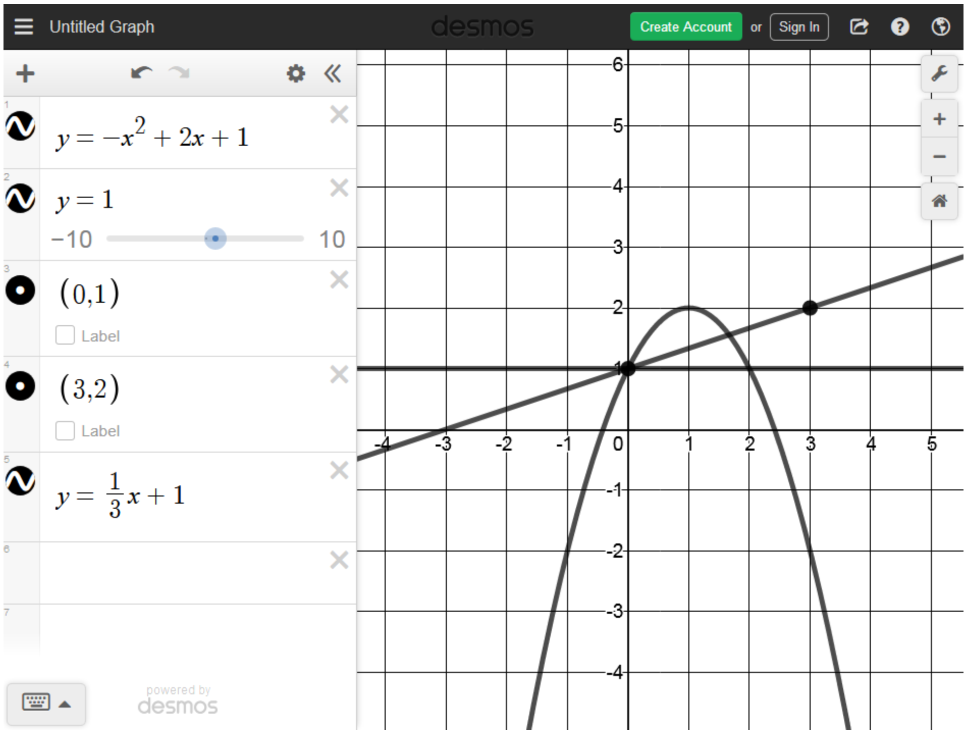


Ask, “What else can you tell me about that line?” Listen for student responses such as horizontal and intersecting the parabola. Whether students raise intersections or not, ask, “Where does this line intersect the parabola?” eliciting that the line will meet the curve at two points. If students haven’t yet mentioned slope, ask what the slope of the line would be and how they know. Then ask, “Suppose I wanted to get this line to angle upward, like this. What would the slope be doing?” As you ask this, gesture with your hand or a pencil to indicate a line that passing through the points (0, 1) and (3, 2). See the model below.

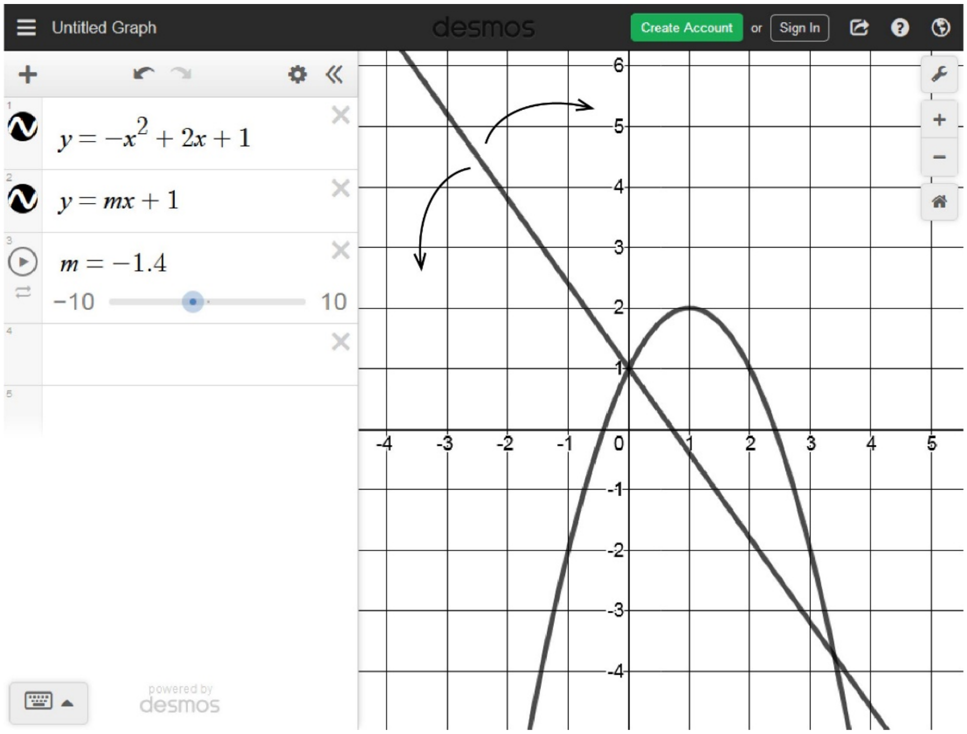


Look for a student who can say that the slope should increase from 0.

Plot the points (0, 1) and (3, 2) on the grid and ask students to find the equation of the line. When students have identified it as $y = \frac{1}{3}x + 1$, plot this line on the grid as well, and have students verify visually that they have found the correct equation.



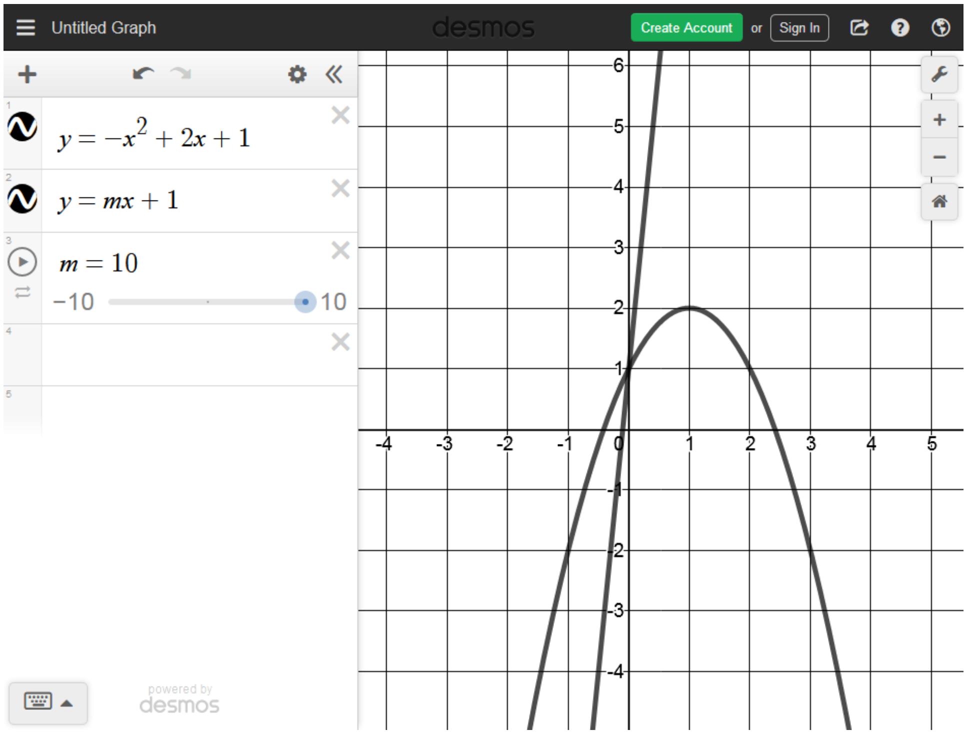
Gesturing with your hand or pencil, again ask students what would happen to the slope of the line if you angled it up to the right a bit more. After students confirm the slope would keep increasing, switch to the second browser tab in which the equation $y = mx + 1$ is graphed with a slider. Give students a moment to verify that we are dealing with the same quadratic function and that this slider will show lines passing through the point (0, 1). Slide the slope back and forth slowly from –10 to 10 a couple of times. See the model below.

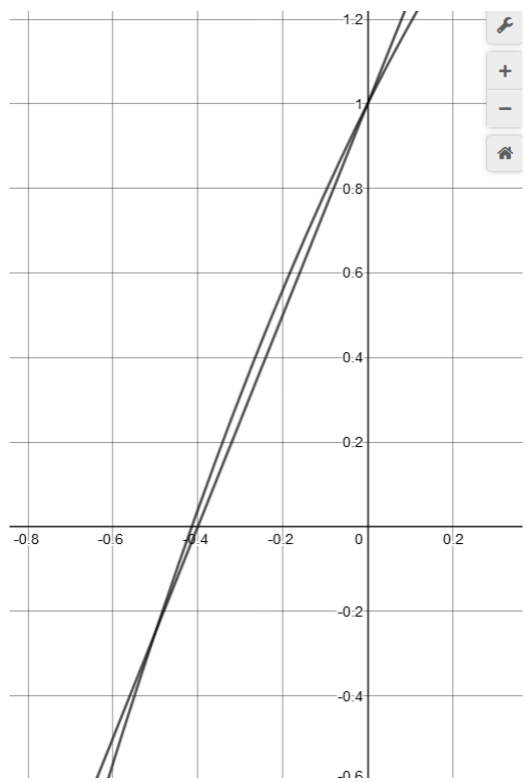


Ask students if any of these lines stands out to them in any way. Look for a student who can say that something interesting is happening when the slope of the line is near 2.

> Spoiler Alert: It's possible that, at this resolution, lines with slopes between 1.5 and 2.5 will all look like they intersect the parabola only once. We have deliberately set the slider to have a step of 0.5 so that the slope of 2 will stand out, but students may still not be sure. This is even more likely if any of your students have visual impairment. It's okay if they don't immediately recognize that only a slope of 2 will produce a line with a unique intersection point with the parabola. Don't tell students this; we can help them gain evidence for this conclusion by zooming in on the graph, as described below.

Ask students to say more about what's interesting at or near a slope of 2. When a student mentions that the line(s) only intersect(s) the graph of $y = -x^2 + 2x + 1$ at one point, ask, "But what about this line? Doesn't this also only touch the other graph at one point?" while placing the m-slider at a slope of 10. See the model below.





Look for a student who can explain that if we zoom out, we will see the line and the parabola intersecting farther down. Zoom out briefly to verify this.

Ask, "But there must be other lines that only touch the graph at this one point, right? Does anyone see one?" If no student suggests it, offer $y = 2.5x + 1$ and move the m-slider to 2.5. Depending on the scale of your graph at the moment, this will likely look like it only touches the parabola at one point. Look for a student who can suggest that if we zoom in, we can see that this line actually intersects the parabola twice. See the model at right.

If students seem unsure, perform a similar check for a slope of 2.2, though this will require setting the step for the slider to 0.2. Hopefully this will be enough to convince students that no other line but $y = 2x + 1$ will have this special property.

Rabbit Hole Alert: If a student suggests a slope even closer to 2, it is possible that Desmos' graphics will be unable to clearly show the two intersection points between the line and the parabola no matter how much you zoom in. You could show the intersections by altering the scale in only one direction, but this distortion could cause students to believe you are altering the functions. If students raise these closer slopes, it's probably best to tell students that the resolution of the graph isn't quite good enough to see, but we would still have two intersection points.

Return the m-slider to 2 and tell students we have a name for this kind of special line. It is called a *tangent line*. Write "tangent line" on the board and ask, "What do you think is true about a tangent line? What characteristics does it have?" Write down their ideas, arriving at something like the following:

A *tangent line* to a curve is a line that only intersects the curve at one point.

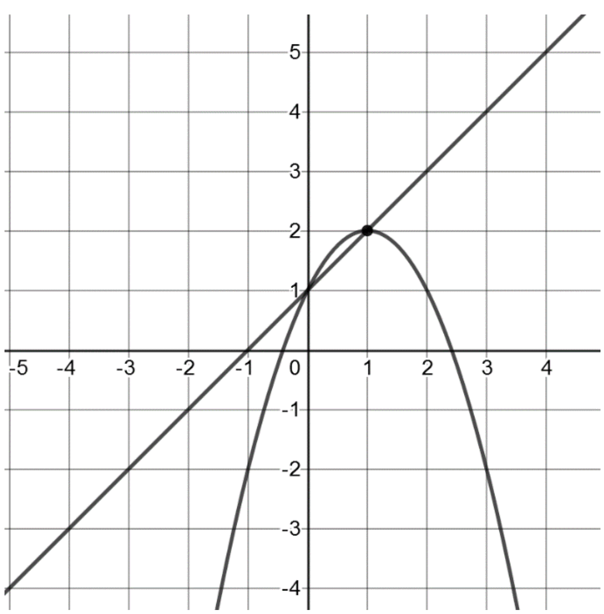
Ask students to write this definition down in their notes.

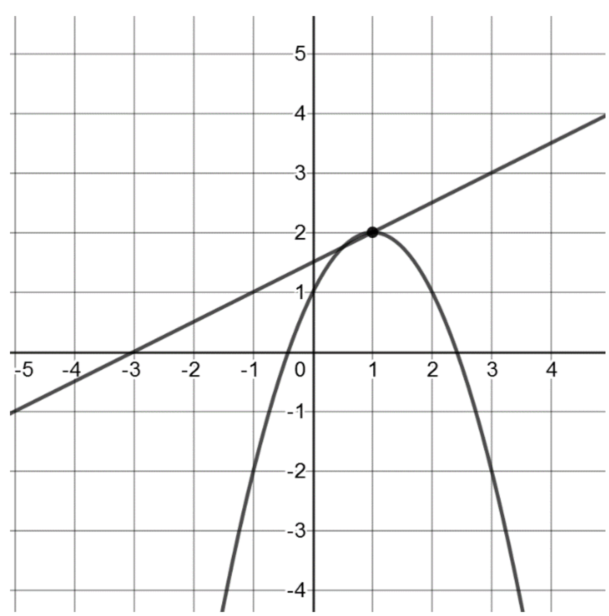
Spoiler Alert: Do not tell students that this definition is inaccurate in many cases. We will explore the issues with this definition and refine it through the following example.

Tell students we're going to practice this idea by considering another point. Enter (1, 2) on a new row so that Desmos will display a point on the parabola at these coordinates. Tell students you want them each to write down the equation of one line that passes through this point. Give students a minute or two to consider this on their own. Ask them not to call out their answers, and let them know that we want to find at least three different lines. As they work, briefly walk around to see what they write down.

When students are ready, call on a few students to hear what equations they came up with. Graph each of these equations in turn to confirm it passes through (1, 2), and for each one, ask, "Is this a tangent line? Why or why not?" Ensure you consider some lines that are not tangent lines before graphing $y = 2$, seeing if students can explain why they wouldn't work. Good options include

$y = x + 1$ or $y = \frac{1}{2}x + \frac{3}{2}$ and See models for these lines below.





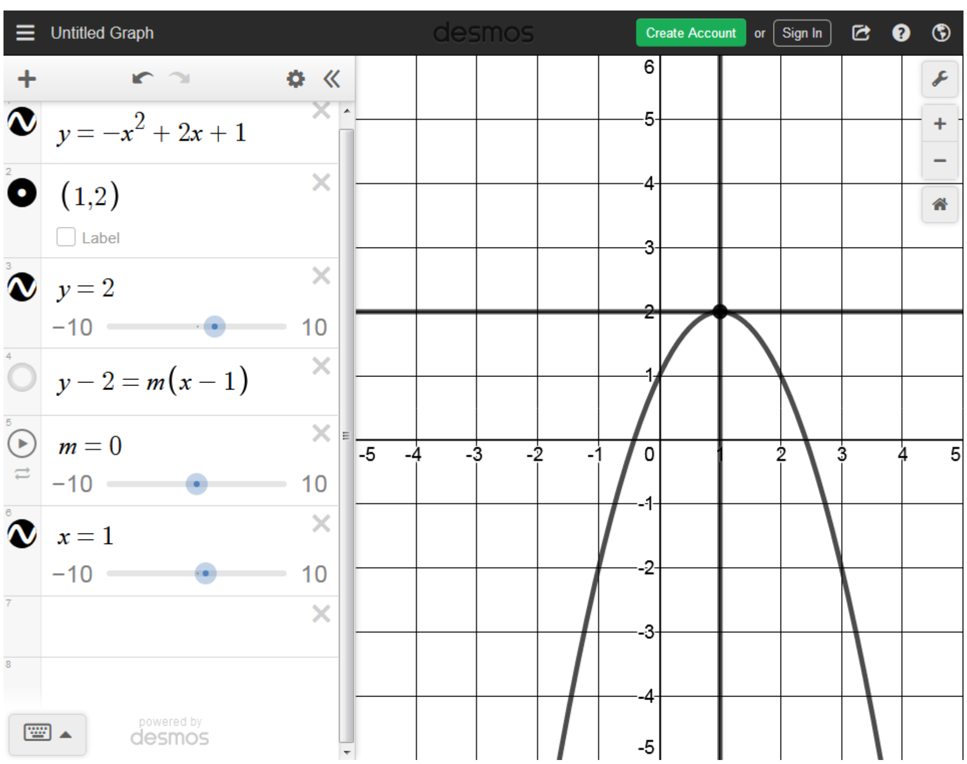
You can also enter these equations into Desmos to verify that they each intersect the parabola at two points. Eventually, have students conclude that $y=2$ is the equation of a tangent line to the parabola through this point.

If no student has yet suggested a vertical line, ask, "Is there any other way I could draw a line that will intersect the parabola at (1, 2) and not at any other point?" Pause here to see if a student will suggest a vertical line. If it does not come out, switch to the tab that includes the slider $y-2=m(x-1)$. As before, give students a moment to verify that the parabola is still the same and that the slider will display lines passing through (1, 2). Ask students to remind you why none of these lines will work except for $y=2$, zooming out as necessary.

Ask, "What if I kept going though?" Set the boundaries on the slider to –50 and 50 and zoom out again as necessary so that students can see the two intersection points of each line with the parabola. Ask, "So is there any other way I could get this line to only touch the parabola at (1, 2)?" Again pause and see if a student will suggest a vertical line.

If no student suggests a vertical line, offer it as a possibility. Ask, "What would be the equation of a line that went straight up the middle of the parabola and didn't lean toward one side or the other?" When a student suggests $x=1$, test this in Desmos. Zoom out and move the grid so that students can see this line will also only intersect the graph of $y=-x^2+2x+1$ at this one point.

Deselect the slider in the equation list so that only the parabola, $y=2$ and $x=1$ are displayed. See the model.

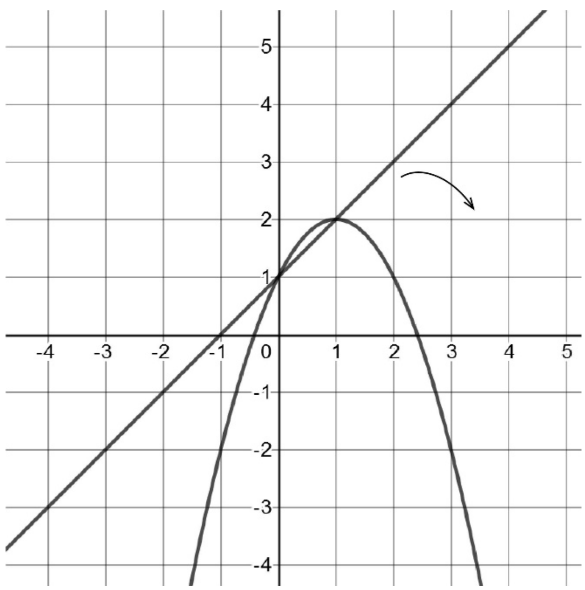


Ask, “So which of these two lines is a tangent line?” Conclude with students that both lines fit our definition.

Tell students that we’re now going to think more about the lines that are *not* tangent lines. Tell students, “I’m going to begin moving the slider. Tell me when the line is *not* a tangent line.” When a student tells you to stop, ask, “Why is that line not a tangent line?” eliciting the two intersection points. Tell students there is a word for these lines as well: *secant lines*. Write the following definition on the board below the definition for tangent line:

A *secant line* to a curve is a line that intersects the curve at least twice.

Reset the bounds on the m-slider for the equation $y-2=m(x-1)$ to –10 and 10 if necessary. Starting with a slope of 10, move slowly toward a slope 0. See the model below.



Ask, “As I do this, what is the other intersection point doing?” Look for a student who can point out that it is getting closer to (1, 2). Ask, “And what are the slopes of these lines doing?” eliciting that the slopes are decreasing. Ask, “What were these lines called again?” to see if students can recall and verbalize the new vocabulary word. Pause the slider at a few key moments and have students confirm the slopes of the corresponding lines. Write these slopes on the board as follows.

3, 2, 1,

Lesson design Note: These slopes are chosen so that the corresponding points of intersection with the parabola have only integer coordinates. This will make it quicker and easier for students to convince themselves of the accuracy of the slopes of the lines, if you’d like them to not only rely on the Desmos display.

Before reaching a slope of 0, quickly swing the slider around to the other side, beginning with a slope of -10. Again ask students what the intersection points are doing as you move the slider, and what the slopes of the secant lines are doing. Here also pause at a few moments and write down the corresponding slopes, so that the board looks something like this:

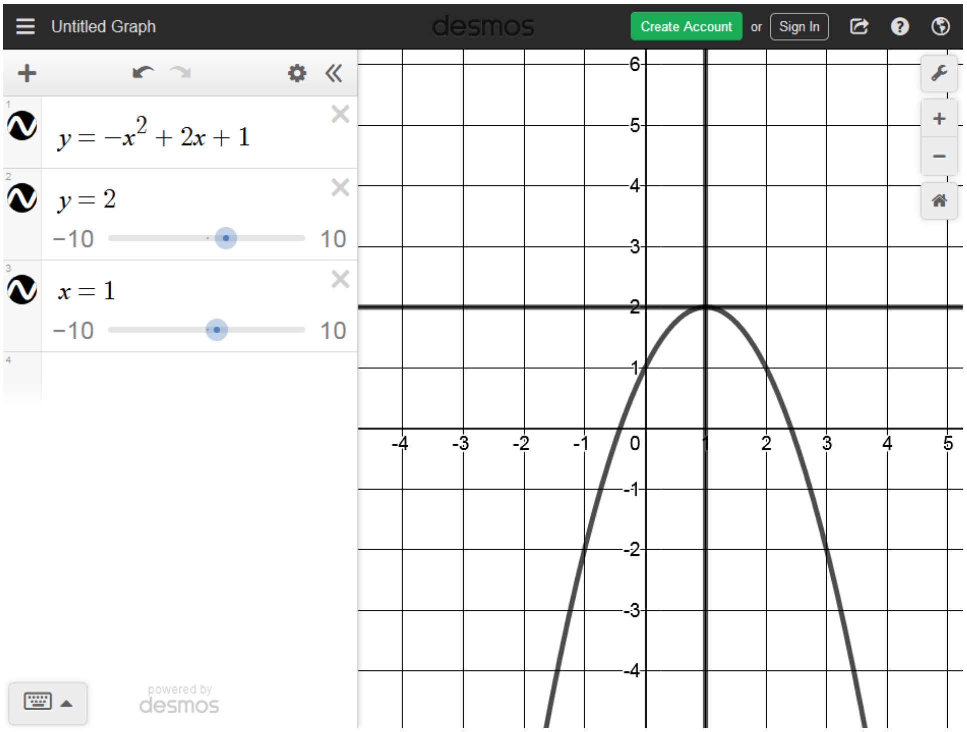
3, 2, 1…   … –1, –2, –3

> Differentiation Note: If you think students will be too distracted by the decimal slopes, consider changing the step on the slider to 1, so that only the above slopes will be displayed as you move the slider.

Remind students that we wish to consider tangent lines, which none of these lines are. Ask, “Have you ever heard the word ‘tangent’ outside of math class?” If students mention only math examples, nudge them again with, “But what about in the real world outside of math?” If no student raises it, ask, “What does it mean if a conversation ‘goes off on a tangent’?” And then, “How do tangents in conversations usually happen?” Elicit from students that tangents often develop naturally from the existing topic, rather than resulting from abrupt changes.

To help reach this conclusion, you could consider saying, “Suppose you’re talking with your friends about which pizza place in your neighborhood is best. How likely is it that in the middle of the conversation you would blurt out the word ‘elephants!” and starting talking about all your favorite elephants?” After students admit that’s pretty unlikely, ask, “Does anyone know that that’s called when someone blurts out a word randomly?” Look for a student who can identify this as a “non-sequitur.” Or, if necessary, offer this word yourself and ask if any student has heard it before and can describe it. Then ask, “So if we’re not likely to make a non-sequitur, then what’s a more likely way the pizza conversation could get off topic?” Look for a student who can say it’s more likely the conversation would go off on a tangent by accident when someone says something related to pizza but not exactly on topic, like, “I don’t like pizza, my favorite place is Taco Bell.”

Ask, “Thinking about that idea of a tangent flowing naturally from the current conversation, what would you expect to be the slope of the tangent line to this graph at (1, 2)?” If necessary, ask, “What could the slope be if we want the tangent line to flow naturally from these other nearby lines?” Once students identify the likely slope as 0, look back at the two “tangent” lines we identified earlier. See the model below.



Gesturing at $y = 1$, ask students what its slope would be. Then ask for the slope of $x = 2$, eliciting undefined or infinity. Ask, "So which of these two lines would flow naturally from the other lines that are nearby?" When students identify $y = 1$, ask, "So which of these better fits the idea of a tangent line?" again having students confirm that $y = 1$ fits the pattern of slopes but $x = 2$ does not.

> Differentiation Note: Consider asking students what they might call $x = 2$ in this situation, seeing if a student might call it a "non-sequitur line". You can tell students that this is not a formal mathematical name, but that they can use it in class if they find it helpful. You can also consider telling students that this is formally called a *normal line*, but because we don't wish to overwhelm students with new vocabulary, it might be best to save that until it is more relevant in a future lesson.

Ask, "So then how could we modify our definition of the tangent line so that's clear we're talking about $y = 1$ and not $x = 2$? Listen and write down student ideas, arriving at something like this on the board.

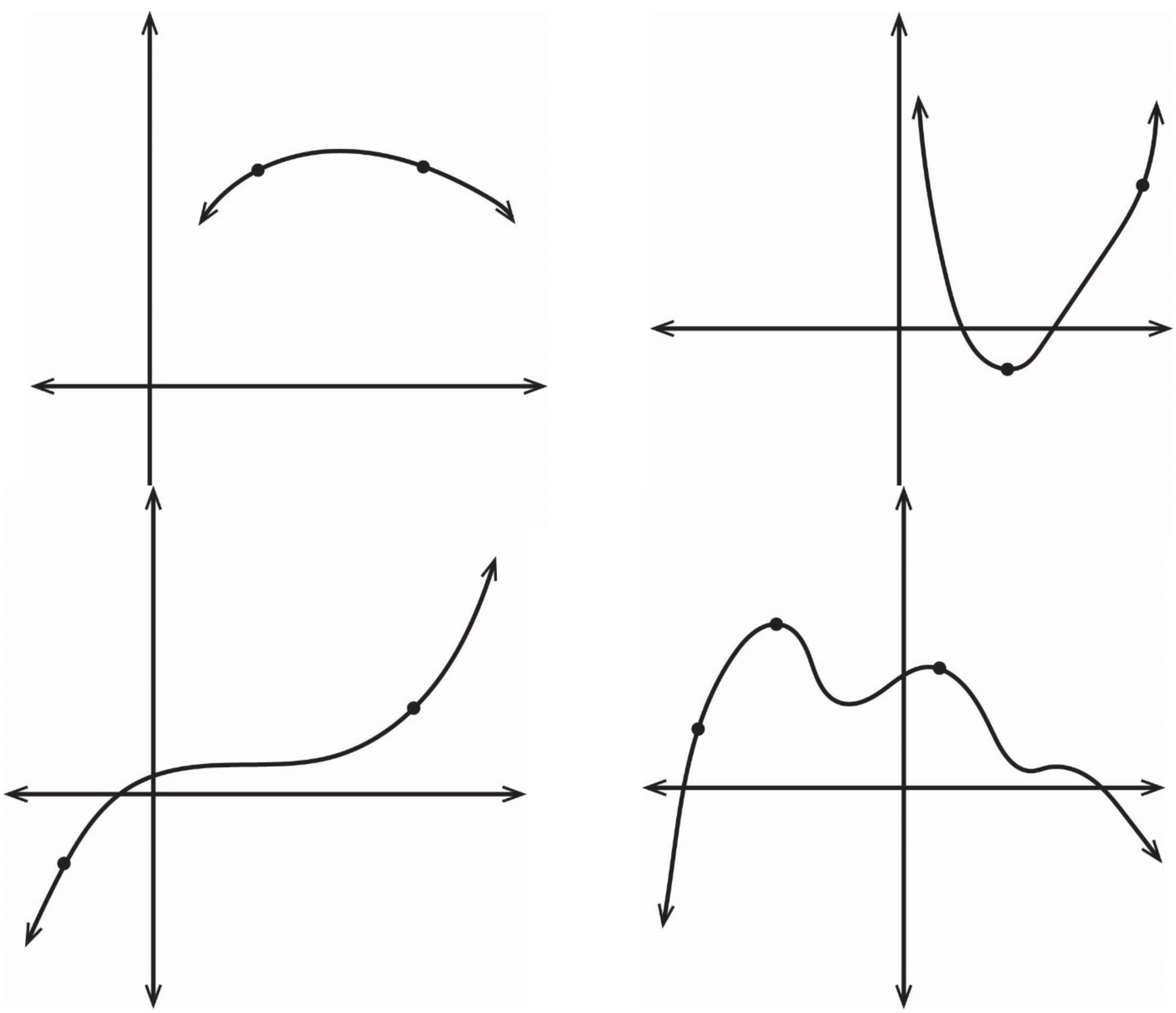
> A *tangent line* to a graph is a line that only intersects the graph at one point and whose slope flows naturally from the slopes of nearby secant lines.

Ask, "So we talked about these slopes as the other intersection point gets closer to (1, 2). Have we seen any other math concepts where we look at what happens as one number or point gets closer to another?" If no student thinks of anything, it can be okay to have them flip back in their notes until someone points out that this is similar to our idea of limits. Ask, "Do you think we could use the word limit in our definition?" Once again write down student ideas, arriving at something like:

A *tangent line* to a graph is a line that only intersects the graph at one point whose slope is the limit of the slopes of nearby secant lines.

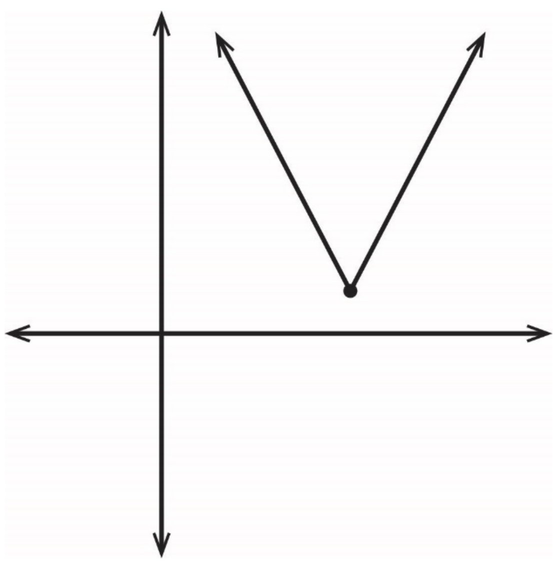
Spoiler Alert: Because this definition is being developed gradually from examples and student ideas, it is still a bit inaccurate, but as before, do not tell students this. In particular, don't mention that tangent lines may sometimes intersect the graph at another point or mention anything about "locally" having one intersection point. Students can again discover this via an example.

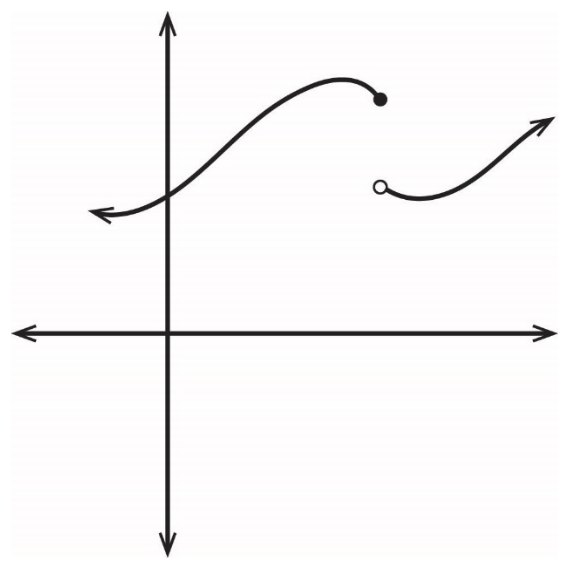
Tell students that before they practice this new concept on their own, it would be good to get some hands-on practice with what tangent lines to curves look like. On all available whiteboards or chalkboards in your room, sketch several axes with functions on them. Along the way, mark a number of points, ideally as many points as you have students. The graphs do not have to correspond to any particular equations, but should have a variety of portions that are increasing and decreasing, convex and concave. See the following examples of the types of functions you can sketch.

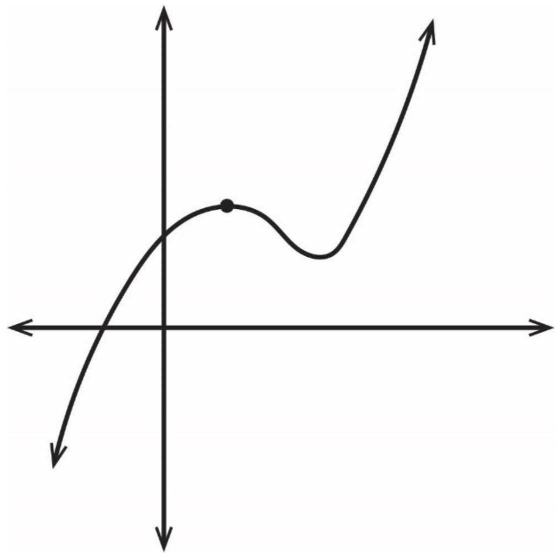


Invite all students to come to the board so they can each draw a little segment of a tangent line to the curve at one of the points you have marked.

> <u>Spoiler Alert</u>: You will want to make sure not to make any sketches that resemble the ones below. At this point, it is best to only give students graphs of functions that are continuously differentiable everywhere. The discussion of what happens when you try to draw a tangent line at a point where the function is not differentiable is of course very interesting, but it would risk distracting students from the concrete understanding of tangent lines and their slopes that we wish to build at this time. There will be time to explore differentiability more thoroughly in later lessons. While the third function is of course continuously differentiable, the chosen point could spoil the discovery about a single intersection point that we will eventually be eliminating from our definition of a tangent line.







After all the line segments are drawn, find a way for each student to briefly see every drawing and ask students to evaluate each other's work. This could involve students walking in small groups around the edge of the room or having students cluster at the front of the room, depending on the location of your board space. Students' drawings don't need to be perfect, but look out for any lines that slant in entirely the wrong direction.

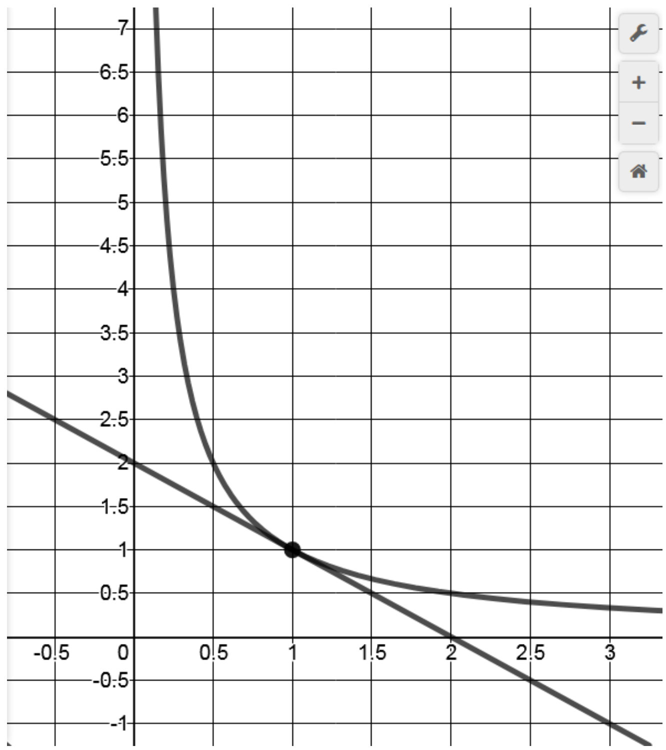
Choosing a tangent line that is reasonably well drawn, ask, "Does this line seem to be a tangent line based on our definition?" When students mention secant lines, draw a couple of secant lines and ask students to approximate the slopes of both the secant lines and the proposed tangent line. It's okay if their answers are imprecise. Ask, "With those approximate slopes, does it seem reasonable that this tangent line's slope would result from the limit of these secant line slopes?"

Distribute, "Two Kinds of Lines, practice".

Give students time to work on the handout individually or in small groups. Circulate as they work, asking them questions occasionally to ensure they understand the answers they are writing. If student seem unsure how to begin question 2, consider suggesting they try to sketch some of the lines, perhaps making a table of values if that would help them. You can also look out for interesting examples of student work or common confusions you might like to draw out in the discussion that follows.

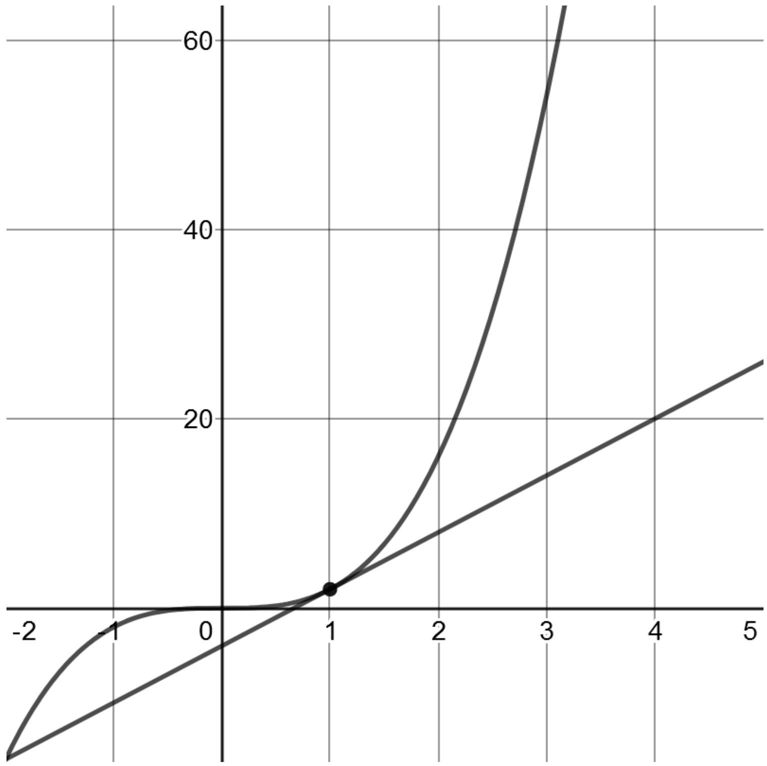
After a number of students have finished the handout, lead a discussion of their answers. If you have access to a document camera, this is a great place to display some student work, especially if they made sketches of secant and tangent lines on the graphs provided. On questions 2 and 3, be sure to draw out why some of the incorrect answers are not tangent lines. Students may point out multiple intersection points with the graph. Or they may identify some of the lines as "non-sequiturs". On questions 4 and 5, many students may be inclined to leave the lines for the written explanation blank, thinking that math is only about number answers. But you can show students you value these explanations by making time for a few students to read their written explanations aloud, asking other students if they seem reasonable. Listen for comments about how the predicted slopes would "flow naturally" from the slopes of the nearby secant lines, much like a tangent in a conversation.

Once you've finished reviewing the handout, tell students that we can also confirm our answers to some of these questions using a graphing calculator like Desmos. Ask students to look back at question 4. Open a new page in Desmos and graph the function $y = \frac{1}{x}$, the point (1, 1), and the line $y - 1 = -1(x - 1)$. See the model below.



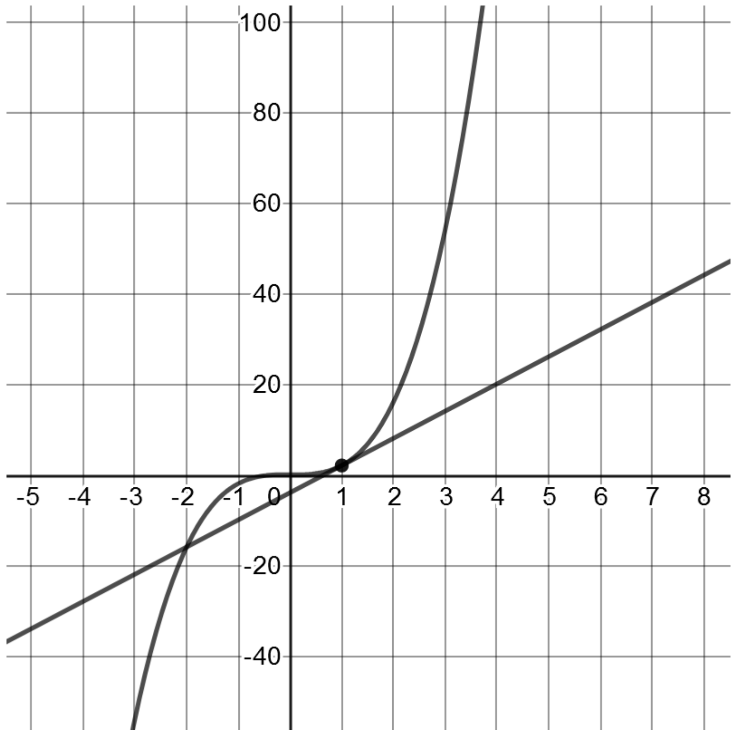
Ask students if it looks like we found the correct slope for the tangent line.

Next ask students to look again at question 5, and perform a similar check, graphing $y = 2x^3$, (1, 2), and $y - 2 = 6(x - 1)$, again asking students for visual confirmation of the tangent line. See the model below.



When students confirm that we found the correct tangent line, ask how they know. Only if students don't refer to our definition of a tangent line ask, "*Why, according to our definition, is this a tangent line?*" It's fine if a student reiterates that the slope of 6 flowed naturally from the slopes of the secant lines. Pause here to see if any student will point out that we also said tangent lines only intersect the graph once, but this line looks it might hit the graph twice. If necessary, ask, "*Where does this line intersect the graph of* $y = 2x^3$?" See if a student will suggest zooming out on Desmos so we can see that there are in fact two intersection points: (1, 2) and (–2, –16).

Ask, "Wow, that's weird. Have we seen any other cases that could be like that? Where the tangent line we found might wind up intersecting the graph more than once?" Give students a moment to see if they spot an example. If necessary ask, "What about the other examples in this handout?" See if a student can point out that the like $y = 1$ will also intersect the sin curve more than once. If students seem skeptical of this, consider graphing these equations on Desmos and zooming out as necessary. See the model below.



Tell students to look back at the definition of a tangent line again. Tell them that we need to modify the definition to account for these cases where the line intersects the graph elsewhere, but we still need to include this important observation about the limit of the slopes. Ask, “Does anyone see a way we could modify our definition to make that happen?” See if students can suggest deleting the words “only” and “at one point,” or if necessary, tell them to do so. By the end of the modifications, the board could look something like this:

> A *tangent line* to a graph is a line that ~~only~~ intersects the graph ~~at one point~~ and whose slope is the limit of the slopes of nearby secant lines.

Tell students to do the same in their notes. This can be a nice reminder to them when studying later about how our understanding of tangent lines progressed. If you think students might be confused by this, you could consider having them add a note like, “The tangent line may only intersect the graph at one point, or it may intersect the graph elsewhere as well.”

This concludes our work on “*Two Kinds of Lines, practice*”.

Distribute, “Tanesha’s Terrific Tofu”.

Give students time to work on the page individually or in pairs. Once enough students have completed the handout, conduct a discussion of their answers, displaying student work under a document camera as appropriate. Nudge students as necessary to ensure they are using the problem’s context and not only relying on formal mathematical language. Also ask students how they knew which line was a tangent line to the graph of the function and which was a secant line. If students only talk about the number of intersection points, ask them how we refined our definition of a tangent line. If necessary, ask, “Do all tangent lines to graphs intersect the graphs at only one point?... So then what makes this line a tangent line?" See if the student can describe imagining other secant lines that are close to the tangent line.

This concludes our work on "*Tanesha's Terrific Tofu*".

Tell students we're going to think a bit more about secant lines in general, not focused on one particular function.

Distribute, "Secant Slope Exploration".

Encourage students to work in small groups. This may require asking them to stand up and move their desks closer together so they can have conversations.

Students may struggle quite a bit with this handout, and that's okay. Try to resist the temptation to tell them too much or give away where we're headed. If students seem completely lost, try asking them some open-ended questions to get them thinking, like, "*What do you see here?*" If necessary, get a bit more targeted in your questions, perhaps asking, "*What does slope mean?"*, "*How do you find slope?*", or "*What could be the coordinates of those two points?*" If students are fluent with the slope formula, these prompts may be all they need.

Differentiation note: If students struggle with the formula, they may also find it helpful to organize their thinking in a table like the one below. You can try suggesting a table, but see if the student can fill in all the entries without your help.

x	*y*
a	*f*(*a*)
a + *h*	*f*(*a* + *h*)

Before discussing the handout, display it under a document camera or draw a sketch of the graph on the board so you have a place to take notes as students describe their ideas. In the discussion that follows, our goal is to get something like the following on the board. Again, see how much students can provide, with little prompting from you. If students are hesitant, try calling on specific students if you noticed they had partial or full explanations on their papers.

$$slope\ of\ the\ secant\ line = \frac{f(a+h)-f(a)}{h}$$

Differentiation note: It's possible students won't think to simplify the denominator of this expression, instead offering the following. It can be good to get this on the board before arriving at the final formula.

$$slope\ of\ the\ secant\ line = \frac{f(a+h)-f(a)}{a+h-a}$$

It's also possible some students will then think they can subtract the *a*'s in the numerator. If students have this confusion, consider asking what the function f(x) could be, eliciting a simple example like x^2 and allowing students to describe what the expression would look like in that case. Ask, "*If the numerator were* $(a+h)^2 - a^2$*, could we subtract the a's?*"

Once this slope is on the board, ask, "*Suppose instead of the slope of this secant line, I wanted a formula for the slope of the tangent line at this point (a,f(a)). Does anyone see a way we could do it?*" Give time for students to respond. If they are silent, try asking, "*What do we know about the slope of a tangent line?"* encouraging students to look back at the definition if necessary.

When student recalls or reads that the slope of a tangent line is "the limit of the slopes of nearby secant lines," ask, "*Could we use a limit in this formula to get the slope of the tangent line?*" Again, give students time to respond. Only if they are silent for several seconds, ask, "*Suppose I looked at some other secant lines nearby. What is happening with this other intersection point as I get closer to the tangent line?*" As you ask this, sketch a couple of more secant lines on the board or on the handout under a document camera.

Look for a student can say that the other intersection point is getting closer to (a,f(a)). Ask, "*And what's happening with h?*" Look for a student who can notice that *h* is getting smaller. Ask, "*What would h be when I finally get to the tangent line?*" Sketch the tangent line, and see if a student can explain that *h* would now be zero, because the two points have become the same point. Ask, "*Does that help me in writing a formula for the slope of the tangent line?"* See if at this point a student can suggest that the limit should be as *h* approaches 0, and write the following on the board.

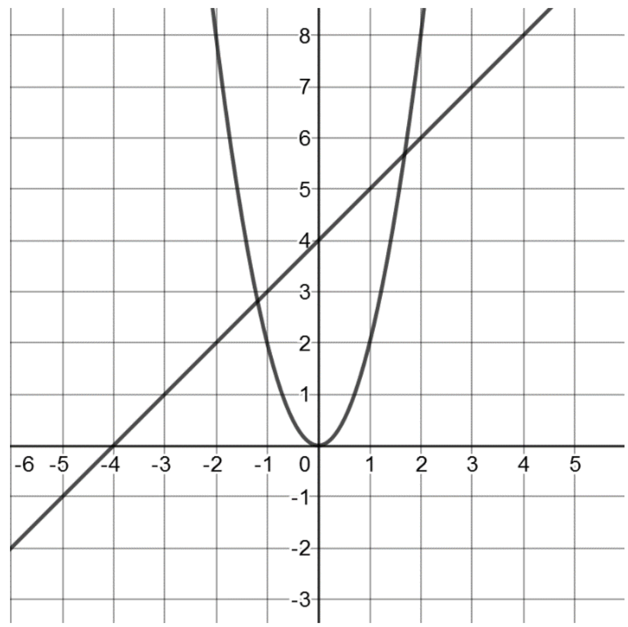
$$slope\ of\ the\ tangent\ line = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Ask students to write this in their notes. This concludes our work on "*Secant Slope Exploration*".

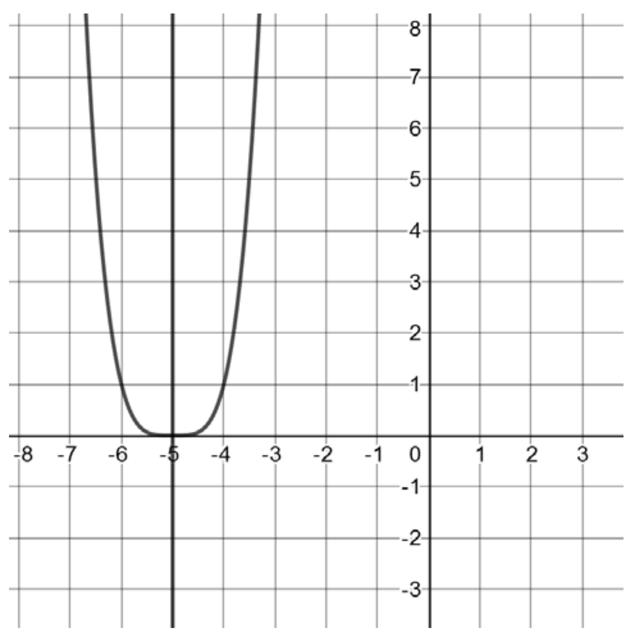
Two Kinds of Lines, practice

1. For each of the four graphs, decide whether the line appears to be a tangent line, a secant line, or neither. Explain your choice.

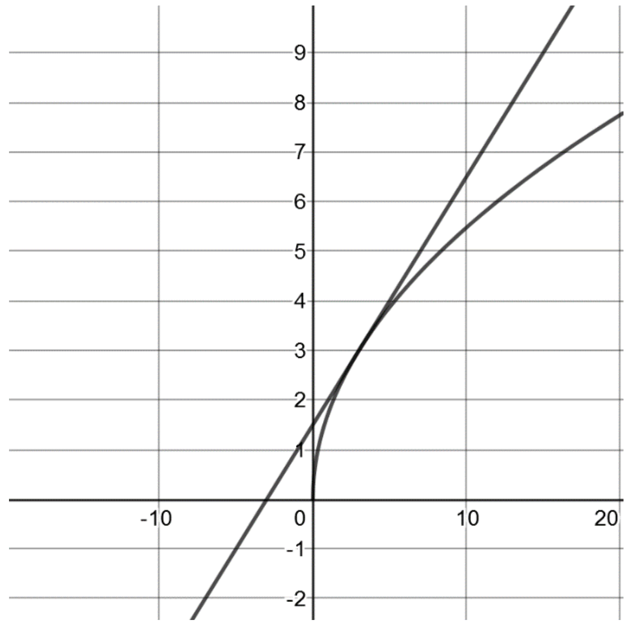
a.



b.



c.



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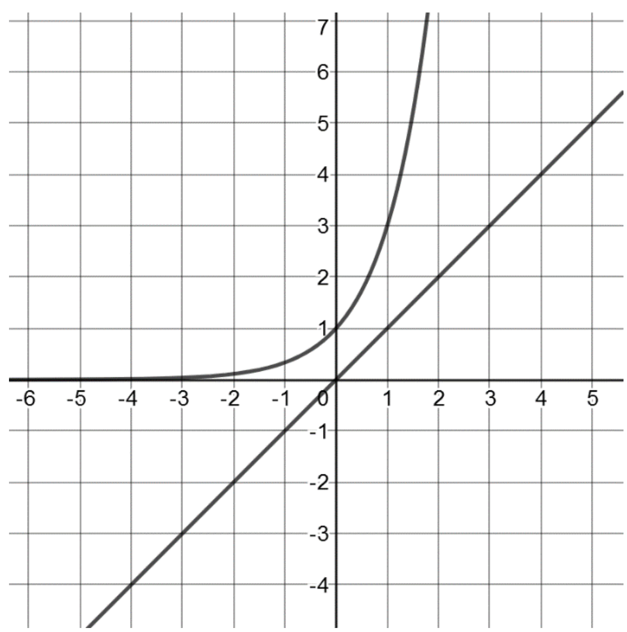
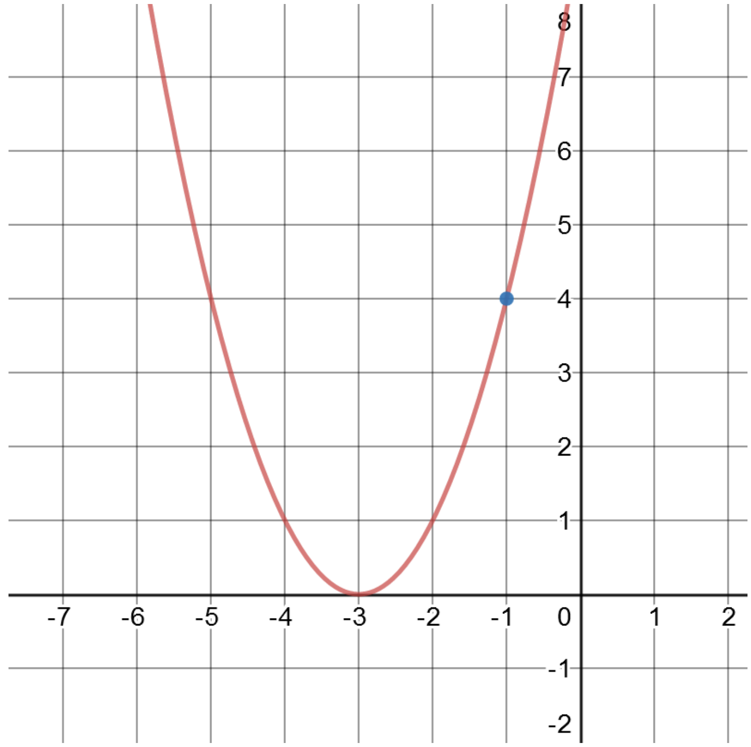


2. Which of the following could be a tangent line to the graph of the function at the point (–1, 4)?



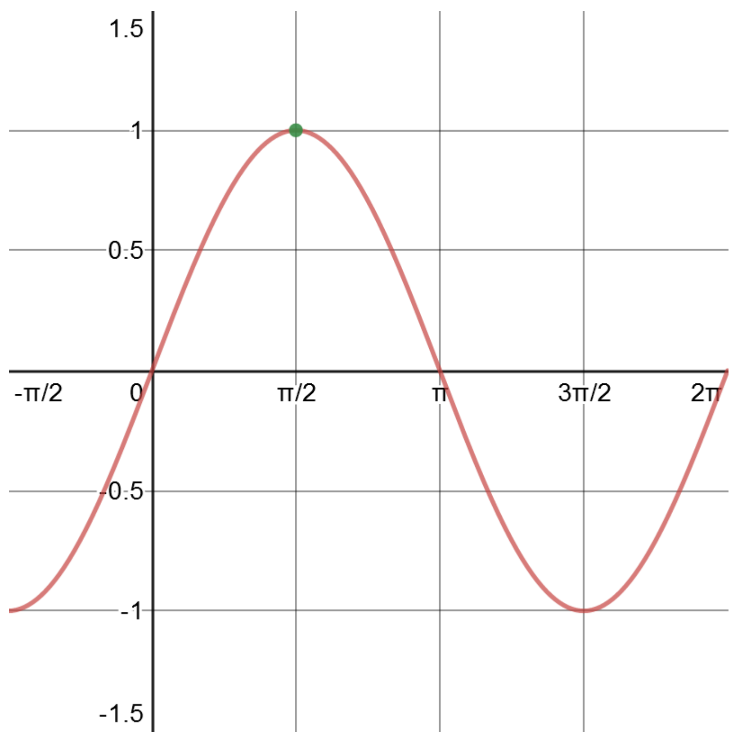
a) $x = -1$

b) $y = 4x + 8$

c) $y = 4$

d) $y - 4 = -\frac{1}{4}(x + 1)$

3. Which of the following could be a tangent line to the graph of the function $y = \sin x$ at the point $\left(\frac{\pi}{2}, 1\right)$?



a) $y = \frac{2}{\pi}x$

b) $y = -1$

c) $y = 1$

d) $x = \frac{\pi}{2}$

4. Using the graphs shown on the opposite page, find the slopes of the four secant lines to the graph of the function $y = \frac{1}{x}$, all of which pass through the point (1, 1).

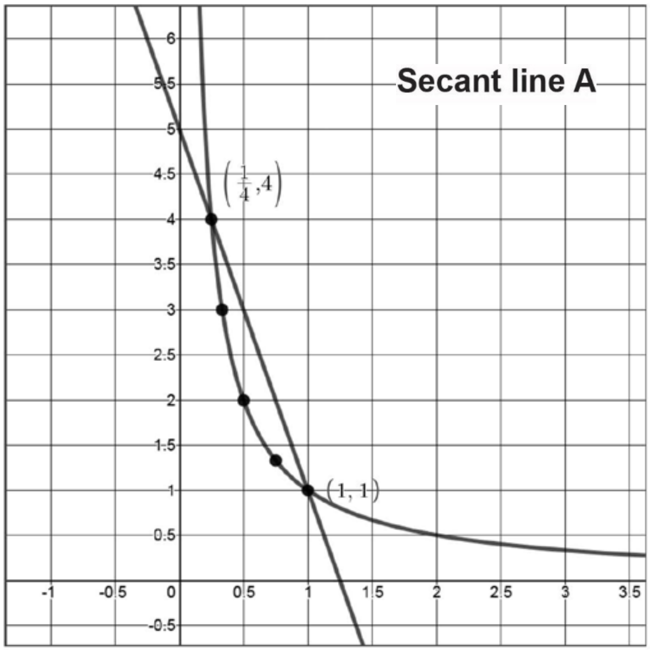
   a) Slope of Secant Line A: ________

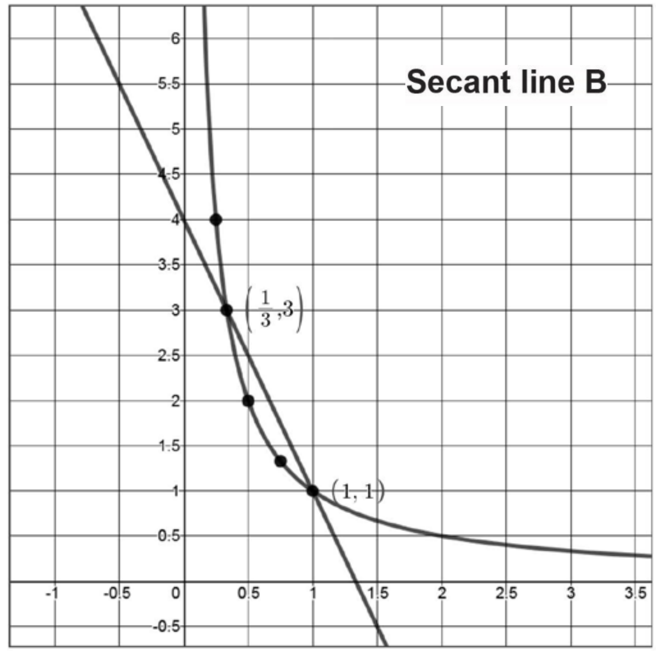
   b) Slope of Secant Line B: ________

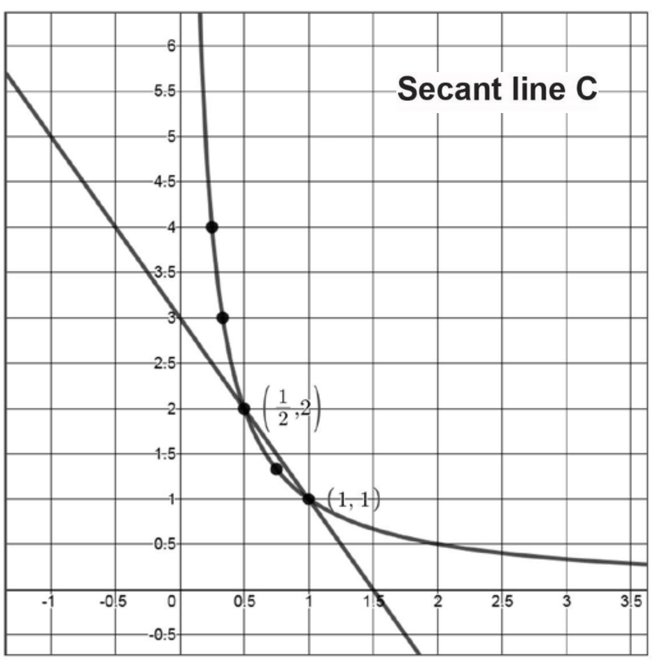
   c) Slope of Secant Line C: ________

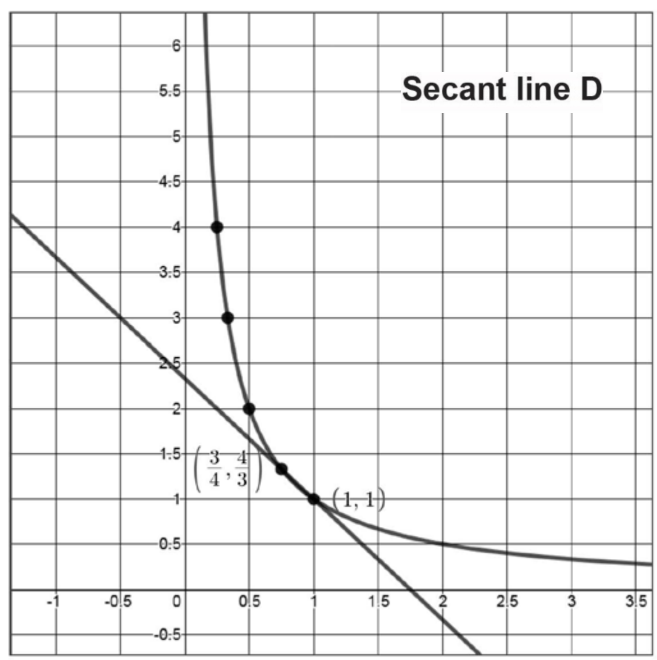
   d) Slope of Secant Line D: ________

   e) What do you think will be the slope of a tangent line to the graph of the function $y = \frac{1}{x}$ that passes through the point (1, 1)? Why is this your prediction?

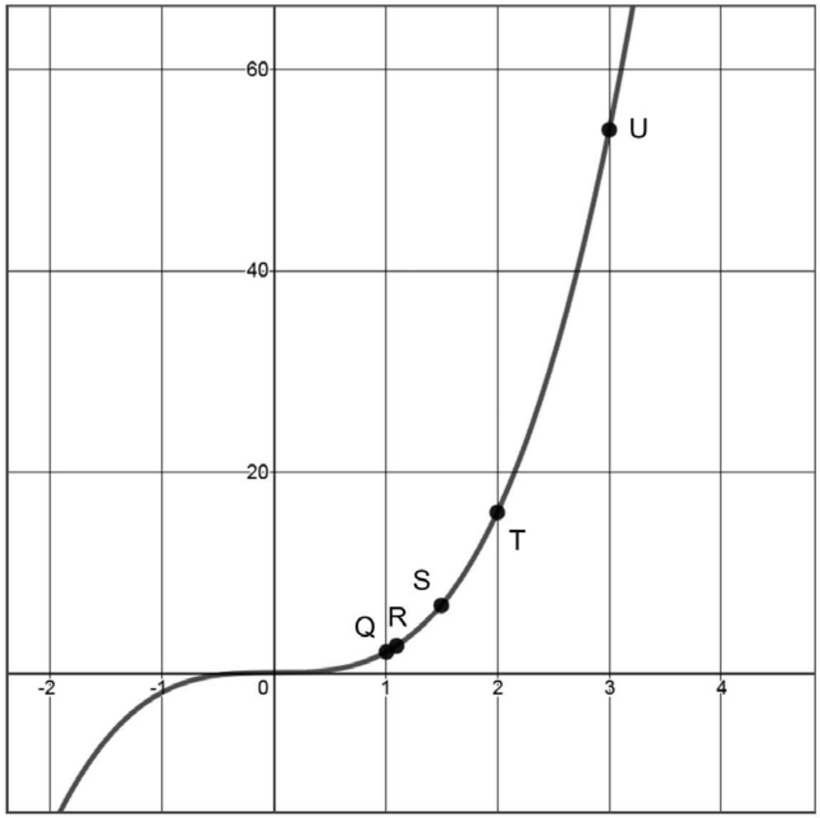

Secant line A
$\left(\frac{1}{4},4\right)$
$(1, 1)$


Secant line B
$\left(\frac{1}{3},3\right)$
$(1, 1)$


Secant line C
$\left(\frac{1}{2},2\right)$
$(1, 1)$


Secant line D
$\left(\frac{3}{4},\frac{4}{3}\right)$
$(1, 1)$

5. Find the slopes of the secant lines to the graph of the function $y = 2x^3$ that pass through the point P = (1, 2) and each of the following points. You may use a calculator.



a) $U = (3, 54)$

b) $T = (2, 16)$

c) $S = (1.5, 6.75)$

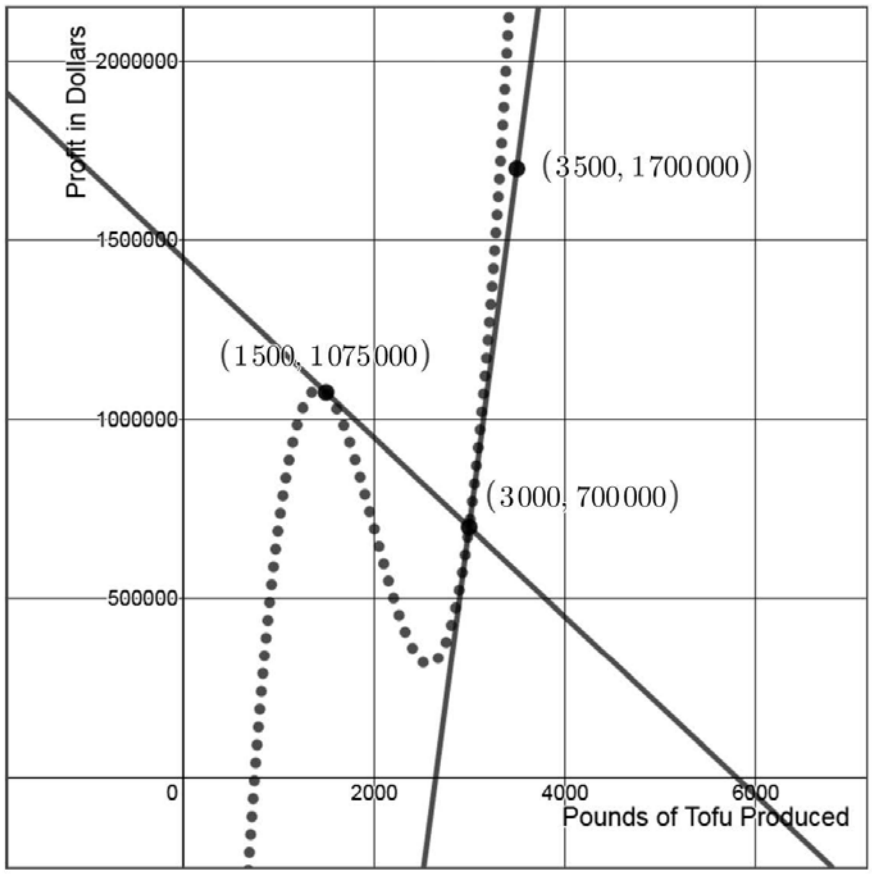
d) $R = (1.1, 2.662)$

e) $Q = (1.01, 2.060602)$

f) What do you think will be the slope of a tangent line to the graph of the function $y = 2x^3$ that passes through the point (1, 2)? Why is this your prediction?

Tanesha's Terrific Tofu

Tanesha's Terrific Tofu supplies tofu to all the organic grocers in a small city. The company's profit in dollars is represented by the function P(x), whose graph is the dotted curve shown below, where *x* represents the number of pounds of tofu produced.



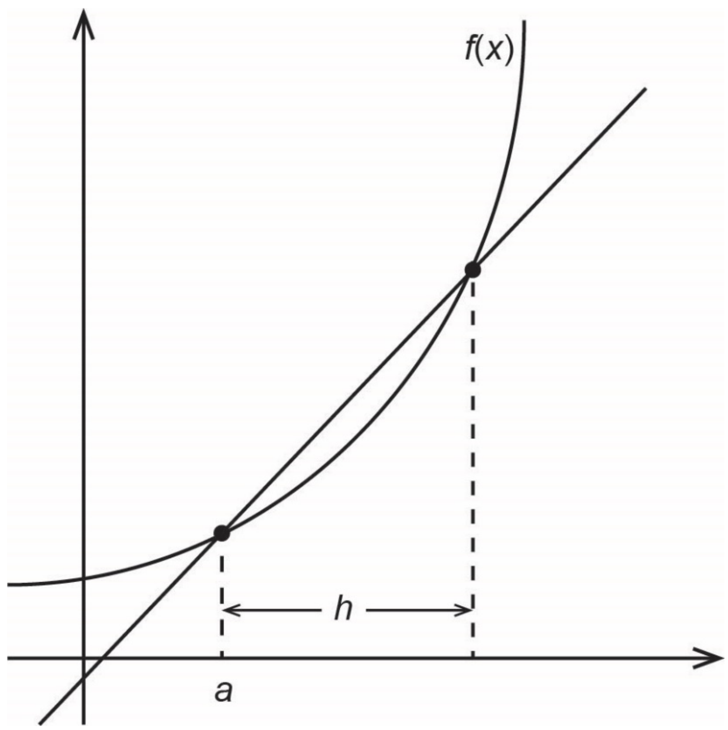
1. Focusing on the dotted curve that shows P(x), why do you think the curve has an interval during which it is decreasing? What could be going on with Tanesha's Terrific Tofu that could explain the behavior of the function over that interval?

2. Find the slope of the secant line to the graph of P(x) shown in the picture.

3. Find the slope of the tangent line to the graph of P(x) shown in the picture.

4. Explain what the two slopes you determined above could mean for Tanesha's Terrific Tofu. Try not to use any formal math language like "*tangent*," "*secant*," or "*slope*". How would you explain these numbers to a friend who has never taken this math course?

Secant Slope Exploration



What do you think would be the slope of this secant line? Why?