a) $f'(x) = 45x^8$

b) $f'(x) = 6^x(\ln 6)$

c) $f'(x) = 0$

d) $f'(x) = -\csc^2(x)$

e) $f'(x) = -\dfrac{1}{\sqrt{1-x^2}}$

f) $f'(x) = 5\cos(5x)$

g) $f'(x) = -\sin(x)$

h) Notice $f(x) = x^{\frac{2}{3}}$ so $f'(x) = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3}\frac{1}{\sqrt[3]{x}} = \boxed{\dfrac{2}{3\sqrt[3]{x}}}$

i) $f'(x) = 1$

j) We have $f(x) = x^{-8}$ so $f'(x) = -8x^{-9} = \boxed{-\dfrac{8}{x^9}}$

k) We need to use logarithmic differentiation

$y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$(\ln y)' = (x\ln x)'$

$\frac{1}{y}y' \underset{\substack{\downarrow \\ \text{product rule}}}{=} 1\cdot \ln x + x\frac{1}{x} = \ln x + 1$

$y' = y(\ln x + 1)$

$f'(x) = \boxed{x^x(\ln x + 1)}$

l) $f'(x) \underset{\substack{\downarrow \\ \text{chain rule}}}{=} 4(12x^3 - 2x^5)^3(12x^3 - 2x^5)' = \boxed{4(12x^3-2x^5)^3(36x^2 - 10x^4)}$

m) $f'(x) = -\sin(x^4)(4x^3)$

n) $f(x) = [\cos(x)]^4$ so $f'(x) = 4[\cos(x)]^3(-\sin x) = \boxed{-4\cos^3(x)\sin x}$

o) $f(x) = x^4 + 4^x + \sin(4) + x^{-1/2} + x^{-4}$ so

$f'(x) = 4x^3 + 4^x(\ln 4) + 0 - \frac{1}{2}x^{-3/2} - 4x^{-5} = \boxed{4x^3 + 4^x(\ln 4) - \dfrac{1}{2\sqrt{x^3}} - \dfrac{4}{x^5}}$

p) $f'(x) \underset{\substack{\downarrow \\ \text{product rule}}}{=} [\sin(x^2+5)]' e^{\cos x} + \sin(x^2+5)(e^{\cos x})' =$

$\cos(x^2+5)(2x)e^{\cos x} + \sin(x^2+5)e^{\cos x}(-\sin x) = \boxed{e^{\cos x}[\cos(x^2+5)2x - \sin(x^2+5)\sin x]}$

q) $f'(x) \underset{\substack{\downarrow \\ \text{quotient rule}}}{=} \dfrac{(5x^2)'\sin x - 5x^2(\sin x)'}{(\sin x)^2} = \boxed{\dfrac{10x\sin x - 5x^2\cos x}{\sin^2 x}}$

r) $f'(x) \underset{\substack{\downarrow \\ \text{chain rule}}}{=} e^{x\sin x}(x\sin x)' \underset{\substack{\downarrow \\ \text{product rule}}}{=} e^{x\sin x}(1\cdot\sin x + x\cos x) = \boxed{e^{x\sin x}(\sin x + x\cos x)}$

s) Notice $f(x) = \frac{1}{5}\cdot x$ so $f'(x) = \boxed{\dfrac{1}{5}}$

t) $f'(x) = \dfrac{(x^2-4)'(x^2-5) - (x^2-4)(x^2-5)'}{(x^2-5)^2} = \dfrac{2x(x^2-5) - (x^2-4)2x}{(x^2-5)^2} =$

quotient rule

$\dfrac{2x^3 - 10x - 2x^3 + 8x}{(x^2-5)^2} = \boxed{\dfrac{-2x}{(x^2-5)^2}}$

u) $f'(x) = (e^{-7x})' \tan(3x) + (e^{-7x})[\tan(3x)]' =$

product rule

$-7e^{-7x}\tan(3x) + e^{-7x}\sec^2(3x)\cdot 3 = \boxed{e^{-7x}[-7\tan(3x) + 3\sec^2(3x)]}$

v) $f'(x) = \dfrac{1}{1+(5x)^2}(5x)' = \boxed{\dfrac{5}{1+25x^2}}$

chain rule

w) $f'(x) = \dfrac{(3x^2+7x+5)'}{3x^2+7x+5} = \boxed{\dfrac{6x+7}{3x^2+7x+5}}$

chain rule

x) $f'(x) = \sec(x)\tan(x)$

(2) We use IMPLICIT DIFFERENTIATION

$(xe^y)' = (5xy + 4y^4)'$ Using product rule and chain rule

$(x)'e^y + x(e^y)' = (5x)'y + (5x)y' + 4(y^4)'$

$e^y + xe^y y' = 5y + 5xy' + 16y^3y'$ Now isolate terms with y' on one side

$xe^y y' - 5xy' - 16y^3y' = 5y - e^y$

$y'(xe^y - 5x - 16y^3) = 5y - e^y$

$\boxed{y' = \dfrac{5y - e^y}{xe^y - 5x - 16y^3}}$ or $\dfrac{e^y - 5y}{5x + 16y^3 - xe^y}$

(3) To find y' we use implicit differentiation

$(x^2 + xy + y^2)' = 0$

$2x + (x)'y + xy' + 2yy' = 0$

$2x + y + xy' + 2yy' = 0$

$y'(x+2y) = -2x - y$

$y' = \dfrac{-2x-y}{x+2y}$

slope $y'(1) = \dfrac{-2-1}{1+2} = \dfrac{-3}{3} = -1$

$y = -x + b$

plug $(1,1)$

$1 = -1 + b$ $b = 2$

$\boxed{y = -x + 2}$

(4) $y' = (5x)' \cos x + (5x)(\cos x)' = 5\cos x - 5x \sin x$

product rule

$y'(\pi) = 5\cos(\pi) - 5\pi \sin(\pi) = -5$

$y = -5x + b$ plug $(\pi, -5\pi)$

$-5\pi = -5\pi + b \quad b = 0$

$\boxed{y = -5x}$