(1) a) -2; b) -3; c) DNE; d) -4; e) No, because $\lim_{x \to -3} f(x)$ DNE

f) 2; g) Yes because $\lim_{x \to 0} f(x) = f(0)$

(2) $\lim_{x \to 2} (5f(x) - 2g(x)) = 5 \lim_{x \to 2} f(x) - 2 \lim_{x \to 2} g(x) = 5(4) - 2(-3) = \boxed{26}$

(3) a) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{5(x+h)^2 - 2(x+h) + 4 - (5x^2 - 2x + 4)}{h} =$

$\lim_{h \to 0} \dfrac{5(x^2 + 2xh + h^2) - 2x - 2h + 4 - 5x^2 + 2x - 4}{h} = \lim_{h \to 0} \dfrac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{2x} - 2h + \cancel{4} - \cancel{5x^2} + \cancel{2x} - \cancel{4}}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{h}(10x + 5h - 2)}{\cancel{h}} = \lim_{h \to 0} (10x + 5h - 2) = \boxed{10x - 2}$

b) slope $= f'(-1) = 10(-1) - 2 = -12$

line $y = -12x + b$ point $(-1, 11)$

$11 = -12(-1) + b$ $\quad 11 = 12 + b \quad b = -1$ $\quad \boxed{y = -12x - 1}$

(4) a) 6

b) $3(-2)^2 - 7(-2) - 4 = 12 + 14 - 4 = \boxed{22}$

c) $\lim_{x \to 3} \dfrac{x-3}{x-3} = \dfrac{0}{0}$ (common factor) $= \lim_{x \to 3} 1 = \boxed{1}$

d) $\lim_{x \to 3} \dfrac{x-3}{x+3} = \dfrac{0}{6} = \boxed{0}$

e) $\lim_{x \to 3} \dfrac{x+3}{x-3} = \dfrac{6}{0} = \boxed{\text{DNE}}$

f) $\lim_{x \to -3} \dfrac{x+3}{x-3} = \dfrac{0}{-6} = \boxed{0}$

g) $\lim_{x \to \infty} \dfrac{x^2+3}{x-3} = \boxed{\text{DNE}}$ (infinity because the degree of the numerator is bigger than the degree of the denominator)

h) $\lim_{x \to \infty} \dfrac{x+3}{x-3} = \boxed{1}$ $\quad \dfrac{1 \cdot x}{1 \cdot x} = 1$ is the limit because numerator and denominator have the same degree

i) $\lim_{x \to \infty} \dfrac{x+3}{x^2-3} = \boxed{0}$ because the degree of the denominator is bigger than the degree of the numerator

j) $\lim_{x \to -\infty} \dfrac{x - 6x^2 - 3}{x^2 - 3} = \dfrac{-6}{1} = \boxed{-6}$ because numerator and denominator have the same degree

⑤ $f(x) = \dfrac{2x^2 - 32}{x^2 - 2x - 8} = \dfrac{2(x^2-16)}{(x-4)(x+2)} = \dfrac{2(x-4)(x+4)}{(x-4)(x+2)}$

a) f is continuous where denominator is not 0 $\quad (-\infty, -2) \cup (-2, 4) \cup (4, \infty)$

b) $\lim\limits_{x \to 2} f(x) = f(2) = \dfrac{8-32}{4-4-8} = \dfrac{-24}{-8} = 3$

c) horizontal $\quad \lim\limits_{x \to \infty} f(x) = 2 \quad$ because numerator and denominator have the same degree $\dfrac{2x^2}{x^2} = 2$

$\lim\limits_{x \to -\infty} f(x) = 2 \quad \boxed{y=2 \text{ is the horizontal asymptote}}$

vertical:

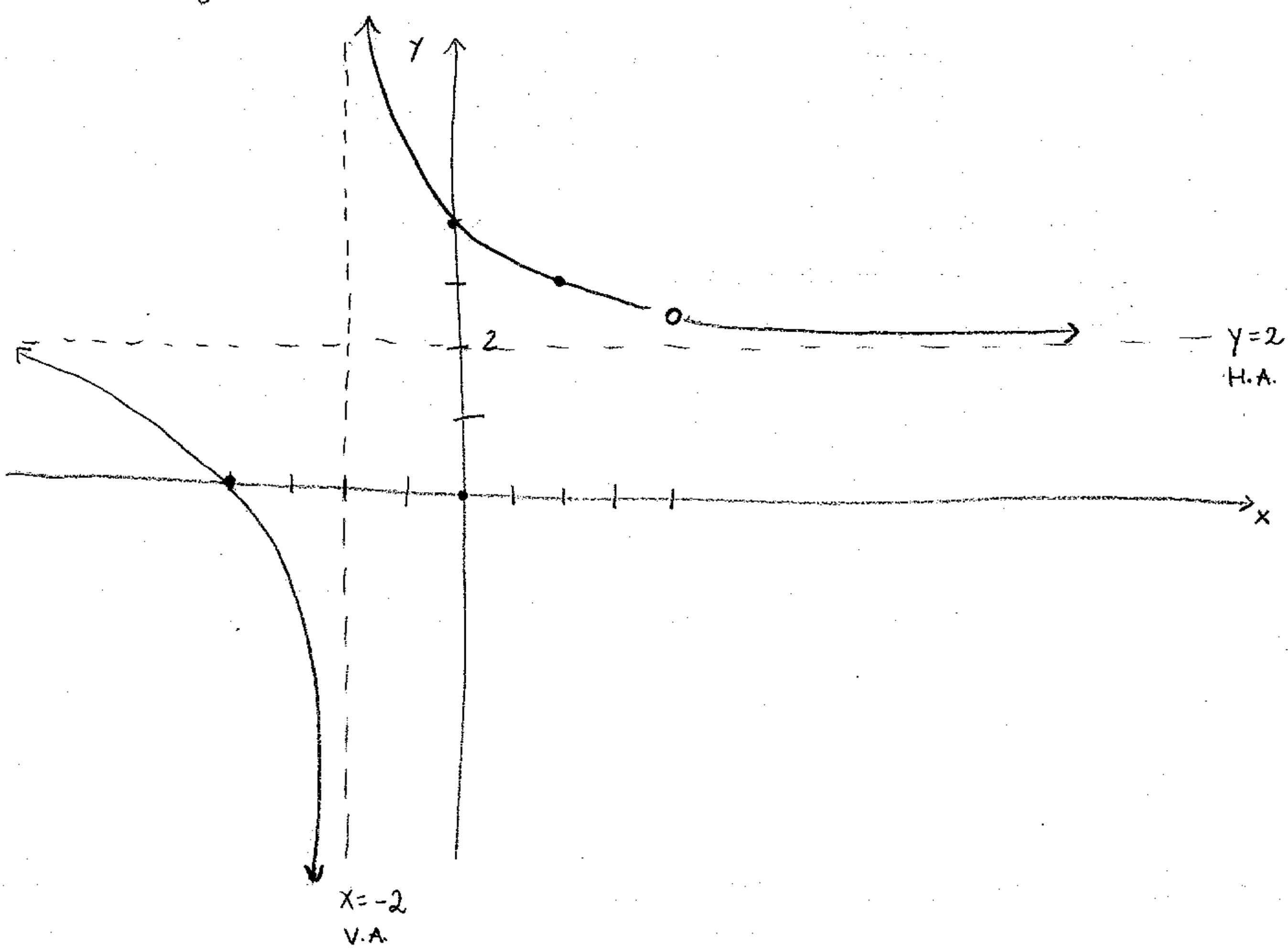
points to check: $x = 4$
$x = -2$

$\lim\limits_{x \to -2} f(x) = \dfrac{-24}{0}$ DNE infinite limit $\quad \boxed{x=-2 \text{ vertical asymptote}}$

$\lim\limits_{x \to 4} f(x) = \lim\limits_{x \to 4} \dfrac{2(x+4)}{(x+2)} = \dfrac{16}{6} = \dfrac{8}{3}$ $\quad$ not an asymptote

$\frac{0}{0}$ $\quad \boxed{\text{HOLE}}$

d)



(6) $f(x) = \dfrac{3}{4-x^2}$

HORIZONTAL $\lim\limits_{x\to\infty} \dfrac{3}{4-x^2} = \lim\limits_{x\to-\infty} \dfrac{3}{4-x^2} = 0$ because the degree of the numerator is smaller than the degree of the denominator

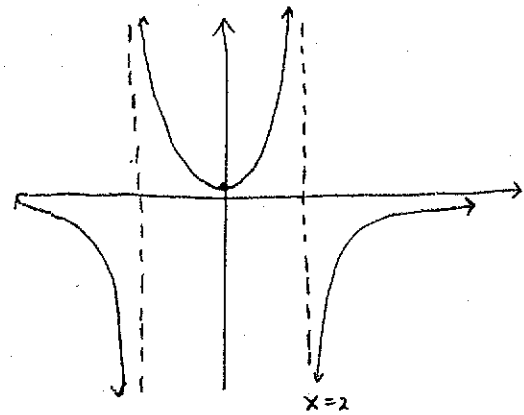
$\boxed{y=0 \text{ is H.A.}}$

VERTICAL $4-x^2=0 \quad x^2=4 \quad x=\pm 2$

$\lim\limits_{x\to 2} \dfrac{3}{4-x^2} = \dfrac{3}{0}$ infinite limit so $\boxed{x=2 \text{ is V.A.}}$

$\lim\limits_{x\to -2} \dfrac{3}{4-x^2} = \dfrac{3}{0}$ infinite limit so $\boxed{x=-2 \text{ is V.A}}$

(You can graph the function to check)



(7) $f(x) = \begin{cases} x^3-x & x<1 \\ x-2 & x\geq 1 \end{cases}$

$\lim\limits_{x\to 0} f(x) = \underset{\text{formula 1}}{0^3-0=0} \qquad f(0) = \underset{\text{formula 1}}{0^3-0=0}$

f is continuous at $x=0$ because $\lim\limits_{x\to 0} f(x) = f(0)$

$\lim\limits_{x\to 1^-} f(x) = \underset{\text{formula 1}}{1^3-1=0} \qquad \lim\limits_{x\to 1^+} f(x) = \underset{\text{formula 2}}{1-2=-1}$

$\lim\limits_{x\to 1} f(x)$ DNE so $f(x)$ is not continuous at $x=1$

(8) $\lim\limits_{x\to 1^-} f(x) = 1^3-1=0$

$\lim\limits_{x\to 1^+} f(x) = 1+C = f(1)$

$0 = 1+C \quad \boxed{C=-1}$