Extreme Values and the Mean Value Theorem - Handout

1. **Definition**: A function f has an *absolute (or global) maximum* at c if $f(c) \geq f(x)$ for every x in its domain. $f(c)$ is called the *maximum value* of f on its domain. Similarly, f has an *absolute (or global) minimum* at c if $f(c) \leq f(x)$ for every x in its domain. $f(c)$ is called the *minimum value* of f on its domain.

2. The maximum and minimum are called the *extreme values* of f.

3. **Definition**: A function f has a *local (or relative) maximum* at c if $f(c) \geq f(x)$ for every x near c. In other words, $f(c) \geq x$ for every x in some open interval containing c. Similarly f has a *local (or relative) minimum* at c if $f(c) \leq f(x)$ for every x near c.

4. **The Extreme Value Theorem**: If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

5. **Fermat's Theorem**: If f has a local maximum or minimum at c, and if $f'(c)$ exists then $f'(c) = 0$.

6. **The Closed Interval Method**: To find the absolute maximum and minimum values of a continuous function on a closed interval $[a, b]$
   (a) Find the values of f at the critical numbers of f in (a, b).
   (b) Find the values of f at the endpoints.
   (c) The largest value from a. and b. is the absolute maximum. The smallest value from a. and b. is the absolute minimum.

7. **The Mean Value Theorem**: Let f be a function that satisfies the following

   (a) f is continuous on $[a, b]$

   (b) f is differentiable on (a, b)

   the there is a number $c \in (a, b)$ such that

   $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

   in other words $f(b) - f(a) = f'(c)(b - a)$.

8. **Corollary** If $f(x)$ is differentiable and $f'(x) = 0$ for all $x \in (a, b)$, then $f(x)$ is constant on (a, b). In other words, $f(x) = C$ for some constant C.

9. **The Sign of the Derivative** Let f be a differentiable function on an open interval (a, b)

   - If $f'(x) > 0$ for $x \in (a, b)$, then f is increasing on (a, b).
   - If $f'(x) < 0$ for $x \in (a, b)$, then f is decreasing on (a, b).

10. We say that $f(x)$ is **monotonic** on (a, b) if it is either increasing or decreasing on (a, b).

11. **First Derivative Test for Critical Points** Assume that $f(x)$ is differentiable and let c be a critical point of $f(x)$. Then

    - $f'(x)$ changes from + to - at $c \rightarrow f(c)$ is a local maximum.
    - $f'(x)$ changes from - to + at $c \rightarrow f(c)$ is a local minimum.