L'Hôpital's Rule - Handout/Worksheet

1. Consider the limit of a quotient $\lim_{x\to a} \dfrac{f(x)}{g(x)}$. Informally speaking, L'Hôpital's rules states that when $f(x)/g(x)$ has an indeterminate form of type $0/0$, or ∞/∞ at $x = a$, then we can replace $f(x)/g(x)$ by the quotient of the derivatives $f'(x)/g'(x)$.

2. **L'Hôpital's Rule** Assume that $f(x)$ and $g(x)$ are differentiable on an open interval containing a and that $f(a) = g(a) = 0$. Also assume that $g'(x)$ (except possibly at a). Then
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$
if the limit on the right exists or is infinite (∞ or $-\infty$). This conclusion also holds if $f(x)$ and $g(x)$ are differentiable for x near (but not equal to) a and $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$. Furthermore, this rule is valid for one-sided limits.

3. Use L'Hôpital's Rule to evaluate:

   (a) $\lim_{x\to 2} \dfrac{4 - x^2}{\sin(\pi x)}$

   (b) $\lim_{x\to -\infty} \dfrac{7x^2 + 4x}{9 - 3x^2}$