Derivatives

1. **Definition:** The ***the derivative of a function*** f ***at a number*** a denoted by $f'(a)$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists, or equivalently, if we let $x = a + h$, then $h = x - a$ and $h \to 0$ only if $x \to a$ and

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

2. There are two interpretations of the derivative:

   (a) The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a. If we use the point-slope form of the equation of a line:

   $$y - f(a) = f'(a)(x - a)$$

   (b) The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$.

3. If we replace a by a variable x, we obtain

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

So given any number x for which this limit exists, we assign to x the number $f'(x)$. We can regard f' as a new function called the ***derivative of f***.

4. The domain of $f' = \{x | f'(x) exists\}$ and may be smaller than the domain of f.

5. A function f is ***differentiable at*** a if $f'(a)$ exists. It is ***differentiable on an open interval*** (a, b) if it is differentiable at every number in the interval.

6. **Theorem**: If f is differentiable at a, then f is continuous at a. WARNING: The converse is not true!

Differentiation Formulas

1. The derivative of a **constant function**

$$\frac{d(c)}{dx} = 0$$

2. **Power Rule**: If n is a positive integer, then

$$\frac{dx^n}{dx} = nx^{n-1}$$

3. **Constant Multiple Rule**: If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

4. **Sum Rule**: If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

5. **Difference Rule**: If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

6. **Product Rule**: If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

7. **Quotient Rule**: If f and g are both differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$