Limits at Infinity - Handout/Worksheet

1. **Definition**: Let f be a function defined on some interval (a, ∞) or $(-\infty, a)$. Then
$$\lim_{x\to\infty} f(x) = L$$
or
$$\lim_{x\to-\infty} f(x) = L$$
means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large (or small).

2. **Definition**: The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either
$$\lim_{x\to\infty} f(x) = L$$
or
$$\lim_{x\to-\infty} f(x) = L.$$

3. **Important Facts**:

   (a)
$$\lim_{x\to\infty} x^n = +\infty \quad n = 1, 2, 3 \ldots$$

   (b)
$$\lim_{x\to-\infty} x^n = \begin{cases} +\infty & \text{if } n = 2, 4, 6, \ldots, \\ -\infty & \text{if n=1,3,5,}\ldots, \end{cases}$$

   (c)
$$\lim_{x\to\infty} \frac{1}{x^n} = \left(\lim_{x\to\infty} \frac{1}{x}\right)^n = 0$$

   (d) If $c_n \neq 0$ then
$$\lim_{x\to\pm\infty} c_0 + c_1 x + \cdots + c_n x^n = \lim_{x\to\pm\infty} c_n x^n$$

   (e) If $c_n \neq 0$ and $d_n \neq 0$ then
$$\lim_{x\to\pm\infty} \frac{c_0 + c_1 x + \cdots + c_n x^n}{d_0 + d_1 x + \cdots + d_m x^m} = \lim_{x\to\pm\infty} \frac{c_n x^n}{d_m x^m}$$

Calculate:

1. $\lim_{x\to\pm\infty} 3x^5 - 4x^2 + 5$

2. $\lim_{x\to\pm\infty} 3 - 4x^{-3} + 5x^{-5}$

3. $\lim_{x\to\infty} \frac{3x^4 - 7x + 9}{7x^4 - 4}$

4. $\lim_{x\to\infty} \frac{3x^4 - 7x + 9}{7x^3 - 4}$

5. $\lim_{x\to-\infty} \frac{3x^8 - 7x + 9}{7x^3 - 4}$

6. $\lim_{x\to-\infty} \frac{3x^7 - 7x + 9}{7x^3 - 4}$