Continuity

1. A function f is said to be **continuous at a point** c if the following three conditions are satisfied:
   - $f(c)$ is defined
   - $\lim_{x \to c} f(x)$ exists
   - $\lim_{x \to c} f(x) = f(c)$.
2. If one or more of the conditions in this definition fails, we say that f **has a discontinuity at the point** c, or that f **is discontinuous at** c.
3. Geometrically, think of a continuous function at every value in an interval as a **function whose graph has no break in it**. The graph can be drawn without removing your pen from the paper.
4. A function f is said to be **continuous from the right at** c if
   $$\lim_{x \to c^+} f(x) = f(c)$$
   and **continuous from the left at** c if
   $$\lim_{x \to c^-} f(x) = f(c).$$
5. If f is continuous at each point of an interval we say that **f is continuous on the interval**. (We understand continuous at the endpoints to mean continuous from the left (or right)).
6. If f and g are continuous at c and k is a constant then, the following functions are also continuous at c:
   - $f + g$
   - $f - g$
   - kf
   - fg
   - $\dfrac{f}{g}$ if $g(c) \neq 0$.
7. Theorem: a) Any polynomial is continuous everywhere, that is, it is continuous on the set of all real numbers $\mathbf{R}$. b) Any rational function is continuous wherever it is defined; that is, it is continuous on its domain. c) root functions and trigonometric functions are also continuous at every value in their domains.
8. If f is continuous on an interval and if f^{-1} exists, then f^{-1} is continuous.
9. If g is continuous at c and f is continuous at $g(c)$ then the composition $f \circ g$ is continuous at c.