One-Sided Limits and Intro. to Continuity - Worksheet 1.4&1.5

Evaluate the given limits of the piecewise defined functions given.

1.
$$f(x) = \begin{cases} x^2 - 1 & x < -1 \\ x^3 + 1 & -1 \leq x \leq 1 \\ x^2 + 1 & x > 1 \end{cases}$$

(a) $\lim_{x \to 1^-} f(x)$

(b) $\lim_{x \to 1^+} f(x)$

(c) $\lim_{x \to 1} f(x)$

(d) $f(1)$

2.
$$f(x) = \begin{cases} 1 - \cos^2(x) & x < a \\ \sin^2(x) & x \geq a \end{cases}$$

where a is a real number.

(a) $\lim_{x \to a^-} f(x)$

(b) $\lim_{x \to a^+} f(x)$

(c) $\lim_{x \to a} f(x)$

(d) $f(a)$

3.
$$f(x) = \begin{cases} x^2 & x < 2 \\ x + 1 & x = 2 \\ -x^2 + 2x + 4 & x > 2 \end{cases}$$

(a) $\lim_{x \to 2^-} f(x)$

(b) $\lim_{x \to 2^+} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $f(2)$

Determine if f is continuous at the indicated values. If not, explain why.

1.
$$f(x) = \begin{cases} 1 & x = 0 \\ \dfrac{\sin(x)}{x} & x > 0 \end{cases}$$

(a) $x = 0$

(b) $x = \pi$

$$f(x) = \begin{cases} \dfrac{x^2 + 5x + 4}{x^2 + 3x + 2} & x \neq -1 \\ 3 & x = -1 \end{cases}$$

(a) $x = -1$

(b) $x = 10$

Give the intervals on which the given function is continuous.

(a) $x^2 - 3x + 9$

(b) $g(x) = \sqrt{x^2 - 4}$

(c) $f(t) = \sqrt{5t^2 - 30}$

(d) $g(x) = \dfrac{1}{1 + x^2}$

(e) $f(x) = e^x$