Limits

1. **Definition:** Assume that $f(x)$ is defined for all x in a open interval containing c, but not necessarily at c itself. We say that

   $$\textit{\textbf{the limit of }} f(x) \textit{\textbf{ as }} x \textit{\textbf{ approaches }} c \textit{\textbf{ is equal to }} L$$

   if $|f(x) - L|$ becomes arbitrarily small when x is any number sufficiently close to (but not equal) to c. In this case we write,

   $$\lim_{x \to c} f(x) = L.$$

   We also say that $f(x)$ *approaches* or *converges to* L as $x \to c$ (and we write $f(x) \to L$.

2. If the values of $f(x)$ do not converge to any limit as $x \to c$, we say that $\lim_{x \to c} f(x)$ *does not exist.*

3. **NOTICE:** We never consider $x = c$. In fact, $f(x)$ *may not even be defined at* $x = c$.

4. **One-sided Limits** We write

   $$\lim_{x \to c^-} f(x) = L$$

   and say the *left hand limit of* $f(x)$ *as* x *approaches* c (or *the limit of* $f(x)$ *as* x *approaches* c *from the left*). Similarly

   $$\lim_{x \to c^+} f(x) = L$$

   is the *right hand limit of* $f(x)$ *as* x *approaches* c *is equal to* l.

5. We observe the following:

   $$\lim_{x \to c} f(x) = L \quad \Leftrightarrow \quad \lim_{x \to c^-} f(x) = L \quad = \quad \lim_{x \to c^+} f(x) = L.$$

6. **Infinite Limits** Some functions $f(x)$ tend to $+\infty$ or $-\infty$ as x approaches a value c. If so, $\lim_{x \to c}$ does not exist, but we say that $f(x)$ *has an infinite limit* and we write

   $$\lim_{x \to c} f(x) = +\infty$$

   or

   $$\lim_{x \to c} f(x) = -\infty.$$

7. When $f(x)$ approaches $+\infty$ or $-\infty$ as x approaches c from one or both sides the line $x = c$ is called a **vertical asymptote**.