Class #23 - Monday, Nov 25
Section 5.3 & 5.4: Expected Value and Variance of Probability Distributions

---

Definition: The **expected value** of a discrete random variable X which has possible values $x_1, x_2, \ldots, x_n$ is defined as

$$E[X] = \sum_{i=1}^{n} x_i * P(X = x_i)$$

The expected value is sometimes called the *expectation* of X, or simply the *mean* of X, and is usually denoted by μ (the Greek letter "mu").

If X is a random variable with expected value μ, then the **variance** and **standard deviation** of X are defined as follows:

$$\text{Var}(X) = E\left[(X-\mu)^2\right] = \sum_{i=1}^{n} (x_i - \mu)^2 * P(X = x_i)$$

$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

Useful formula for the variance (via some algebra): $\text{Var}(X) = E\left[X^2\right] - \mu^2$

Example 1: On the previous handout, we discussed this hypothetical probability distribution for X = the number of days that it will rain over the next 3 days:

Days of rain, x_i	**Probability** $P(X = x_i)$
0	0.21
1	0.44
2	0.29
3	0.06

Find the expected value, variance, and standard deviation of X.

Example 2: Also on the previous handout, we computed the probability distribution for X = the number of heads observed from flipping a coin three times:

Number of heads, x_i	**Probability** $P(X = x_i)$
0	$1/8 = 0.125$
1	$3/8 = 0.375$
2	$3/8 = 0.375$
3	$1/8 = 0.125$

Find the expected value, variance, and standard deviation of x.