Handout for Section 5.3

Definition of Common Logarithms

Let u and v be real numbers, with $v > 0$. Then

$$\log v = u \qquad \text{exactly when} \qquad 10^u = v.$$

In other words,

log v is the exponent to which 10 must be raised to produce v.

Definition of Natural Logarithms

Let u and v be real numbers, with $v > 0$. Then

$$\ln v = u \qquad \text{exactly when} \qquad e^u = v.$$

In other words,

ln v is the exponent to which e must be raised to produce v.

Properties of Logarithms

Natural Logarithms	Common Logarithms
1. $\ln v$ is defined only when $v > 0$;	$\log v$ is defined only when $v > 0$.
2. $\ln 1 = 0$ and $\ln e = 1$;	$\log 1 = 0$ and $\log 10 = 1$.
3. $\ln e^k = k$ for every real number k;	$\log 10^k = k$ for every real number k.
4. $e^{\ln v} = v$ for every $v > 0$;	$10^{\log v} = v$ for every $v > 0$.

Exponential Functions

Every exponential growth or decay function can be written in the form

$$f(x) = Pe^{kx},$$

where $f(x)$ is the amount at time x, P is the initial quantity, and k is positive for growth and negative for decay.

Domain: all positive real numbers
Range: all real numbers

x-intercept: 1
Vertical Asymptote: y-axis