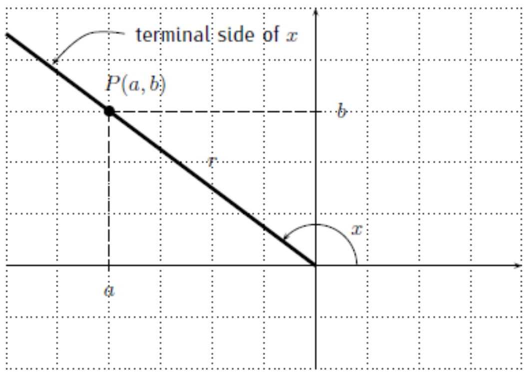
Trigonometry – Useful Formulas and Identities

Definition 17.2. Let x be an angle. Consider the terminal side of the angle x, and assume that the point $P(a, b)$ is a point on the terminal side of x. If r is the distance from P to the origin $(0, 0)$, then we define the **sine**, **cosine**, **tangent**, **cosecant**, **secant**, and **cotangent** as follows:



$$a^2 + b^2 = r^2$$

$$\Longrightarrow \boxed{r = \sqrt{a^2 + b^2}}$$

$$\sin(x) = \frac{b}{r} \qquad \csc(x) = \frac{r}{b}$$

$$\cos(x) = \frac{a}{r} \qquad \sec(x) = \frac{r}{a}$$

$$\tan(x) = \frac{b}{a} \qquad \cot(x) = \frac{a}{b}$$

There are some immediate consequences from the above definition.

$$\boxed{\csc(x) = \frac{1}{\sin(x)}} \qquad \boxed{\sec(x) = \frac{1}{\cos(x)}}$$

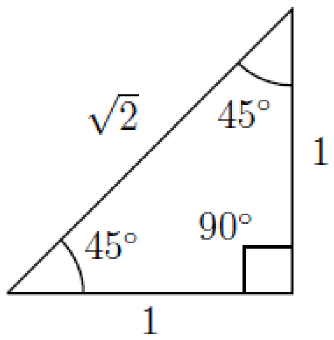
$$\boxed{\tan(x) = \frac{\sin(x)}{\cos(x)}} \qquad \boxed{\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}}$$

$$\boxed{\sin^2(x) + \cos^2(x) = 1} \qquad \boxed{\sec^2(x) = 1 + \tan^2(x)}$$

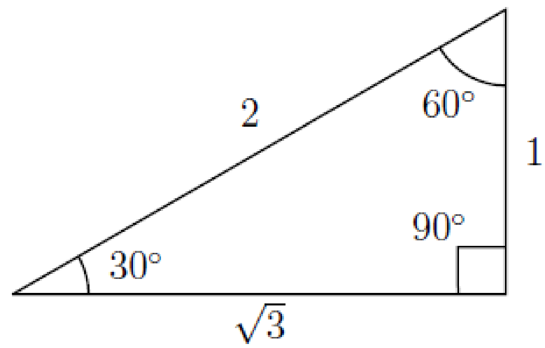
$$\boxed{\sin(x + 2\pi) = \sin(x)} \qquad \boxed{\sin(-x) = -\sin(x)} \qquad \boxed{\sin(\pi - x) = \sin(x)}$$

$$\boxed{\cos(x + 2\pi) = \cos(x)} \qquad \boxed{\cos(-x) = \cos(x)} \qquad \boxed{\cos(\pi - x) = -\cos(x)}$$

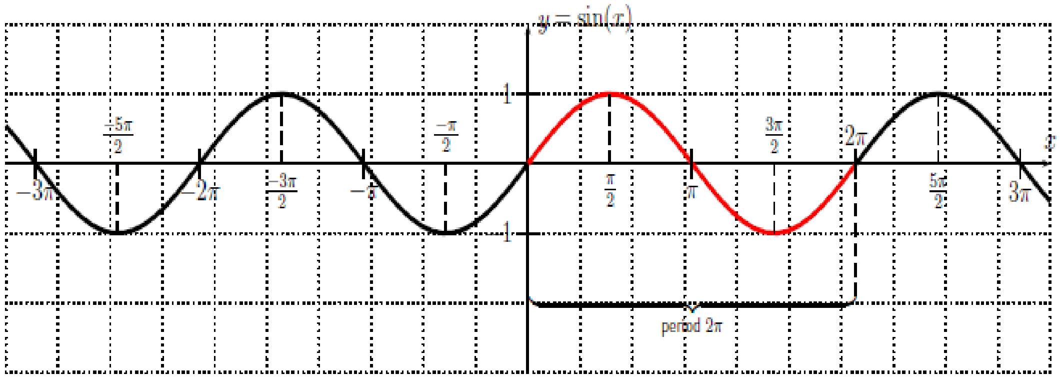
$45° - 45° - 90°$

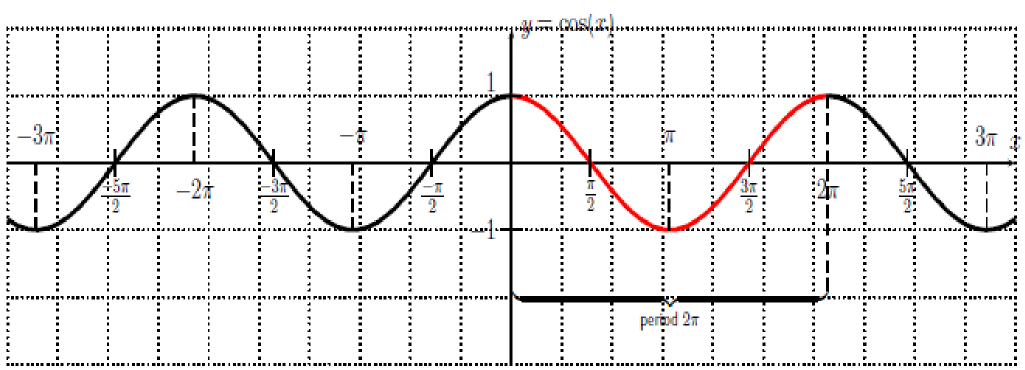


$30° - 60° - 90°$

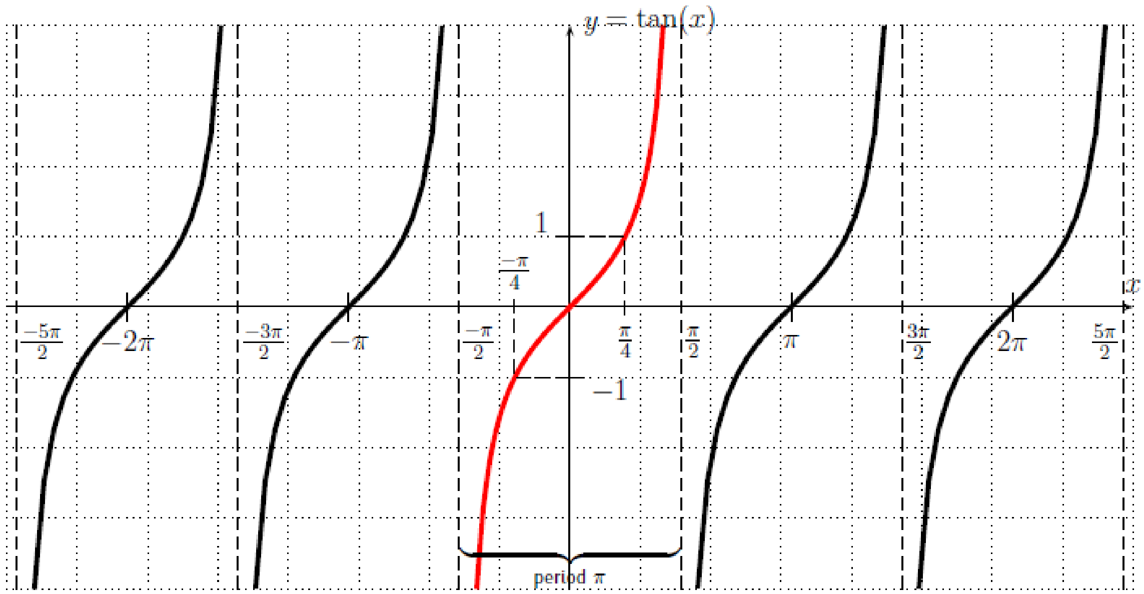


The graph of $y = \sin(x)$ has the following specific values:





Indeed, the graph of $y = \cos(x)$ is that of $y = \sin(x)$ shifted to the left by $\frac{\pi}{2}$.



$y = \tan(x)$
x
1
-1
$\frac{-\pi}{4}$
$\frac{-5\pi}{2}$
-2π
$\frac{-3\pi}{2}$
$-\pi$
$\frac{-\pi}{2}$
$\frac{\pi}{4}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$
period π