Sample Final Exam

1. Determine whether the following argument is valid:

   $p \rightarrow r$,

   $q \rightarrow r$,

   $\neg(p \vee q)$,

   Therefore $\neg r$. Hint: Check if $(p \rightarrow r) \wedge (q \rightarrow r) \wedge \neg(p \vee q) \rightarrow \neg r$ is a tautology.

2. Determine whether the following argument is valid:

   She is a Math Major or a Computer Science Major.

   If she does not know discrete math, she is not a Math Major.

   If she knows discrete math, she is smart.

   She is not a Computer Science Major.

   Therefore, she is smart.

3. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a Venn diagram proof.

4. Determine if the following function is one-to-one and/or onto: $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = 6x^2 + 1$.

5. Give an algorithm (write the pseudocode) that takes a list of n integers $a_1, a_2, \ldots, a_n$ and finds the number of integers each greater than five in the list.

6. Let $f(n) = 3n^2 + 8n + 7$. Show that $f(n)$ is $O(n^2)$. Find C and k from the definition.

7. Use the Euclidean algorithm to find $gcd(34, 21)$.

8. Solve the linear congruence $5x \equiv 3 \mod 11$

9. Use the Principle of Mathematical Induction to prove that $1+4+7+10+\cdots+(3n-2) = \dfrac{n(3n-1)}{2}$ forall $n \geq 1$.

10. Prove the following: Let n be an integer, then n is even if and only if $5n + 4$ is even.

11. Compute a_5 if $a_n = 4 + \lceil 3a_n + 1 \rceil$ when $n > 0$ and $a_0 = 2$.