Worksheet 4.1 & 4.2 - Divisibility and Modular Arithmetic and Integer Representations of Algorithms

1. What are the quotient and remainder when
   (a) 44 is divided by 8?
   (b) 777 is divided by 21?
   (c) -123 is divided by 19?
   (d) -1 is divided by 23?
   (e) -2002 is divided by 87?
   (f) 0 is divided by 17?
   (g) 1,234,567 is divided by 1001?
   (h) -100 is divided by 101?
2. Let m be a positive integer. Show that $a \mod m = b \mod m$ if $a \equiv b(\mod m)$.
3. Evaluate these quantities.
   (a) $-17 \mod 2$
   (b) $144 \mod 7$
   (c) $-101 \mod 13$
   (d) $199 \mod 19$
4. Decide whether each of these integers is congruent to 3 modulo 7.
   (a) 37
   (b) 66
   (c) -17
   (d) -67
5. Find each of these values.
   (a) $(177 \mod 31 + 270 \mod 31) \mod 31$
   (b) $(177 \mod 31 \times 270 \mod 31) \mod 31$
6. Convert the decimal expansion of 100632 to a binary expansion.
7. Convert the binary expansion of each of these integers to a decimal expansion.
   (a) $(11011)_2$
   (b) $(1010110101)_2$

8. Convert the octal and hexadecimal expansion of each of these integers to a binary expansion.

   (a) $(572)_8$

   (b) $(1604)_8$

   (c) $(80E)_{16}$

   (d) $(135AB)_{16}$