Sample Exam #3

1. Write the pseudocode for an algorithm that takes a list of n integers $a_1, a_2, ..., a_n$ and finds the number of integers each greater than five in the list.

2. Write the pseudocode for a brute-force algorithm that finds the largest product of two numbers in a list $a_1, a_2, ..., a_n$ $(n \geq 2)$ that is less than a threshold N.

3. List all the steps used to search for 26 in the list: 4, 6, 7, 13, 16, 20, 26, 31 using

   (a) a linear search

   (b) a binary search

4. Use the bubble sort algorithm to sort 31, 16, 7, 6, 20, show the lists obtained at each step.

5. Use the definition of big$-O$ to prove that $1^3 + 2^3 + \cdots + n^3$ is $O(n^4)$.

6. Let $f(n) = 3n^2 + 8n + 7$. Show that $f(n)$ is $O(n^2)$. Find the witnesses C and k from the definition.

7. Prove that $\dfrac{x^3 + 7x^2 + 3}{2x + 1}$ is $\Theta(x^2)$

8. Find all pairs of functions in this list that are of the same order: $n^2 + \log(n)$, $2^n + 3^n$, $100n^3 + n^2$, $n^2 + 2^n$, $n^2 + n^3$, $3n^3 + 2^n$.

9. Suppose you have two different algorithms for solving a problem. To solve a problem of size n, the first algorithm uses exactly $n\sqrt{n}$ operations and the second algorithm uses exactly $n^2 \log(n)$ operations. As n grows, which algorithm uses fewer operations?

10. For the following questions, find the best big-O notation to describe the complexity of the algorithm. Choose your answers from the following:

$$1, \log_2(n), n, n\log_2(n), n^2, n^3, ..., 2^n, n!.$$

   (a) An algorithm that prints all subsets of size three of the set $\{1, 2, 3, ..., n\}$.

   (b) The number of print statements in the following:

   $i := 1$, $j := 1$ **while** $i \leq n$
       while $j \leq i$
           print hello;
           $j := j + 1$
       $i := i + 1$

   (c) A linear search to find the smallest number in a list of n numbers.

11. Give a big-O estimate for the number of operations (where an operation is an addition or a multiplication) used in this segment of an algorithm:

$t := 0$

for $i = 1$ **to** n

for $j = 1$ **to** n

$t := (it + jt + 1)^2$